

## Rational Subgrouping

### The conceptual foundation of process behavior charts

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While the computations for a process behavior chart are completely general and very robust, the secret to using a process behavior chart effectively lies in the art of rational sampling and rational subgrouping. Rational subgrouping has to do with organizing your data so that the chart will answer the important questions regarding your process.

#### QUESTIONS ASKED BY AVERAGE AND RANGE CHARTS

A key aspect of the efficient use of the process behavior chart is making the charts answer the right questions. In order to do this, the way the data are organized into subgroups must be matched with the structure present in the data. This usually means that each subgroup will be selected from some small region of space, or time, or product, in order to assure relatively homogeneous conditions within the subgroup.

The emphasis on minimizing the variation within the subgroups comes from the fact that it is the variation within the subgroups that is used to compute the limits for the average and range chart. Specifically, the limits depend upon the Average Range, which in turn depends upon the individual subgroup ranges, which measure the variation within the subgroups. Thus, the charts use the *within subgroup variation* to put limits on how much variation should exist *between* the subgroups.

Because of this, the Average Chart asks the question: "Do the subgroup averages vary more than they should, based on the variation within the subgroups?" Or put another way, "Making allowance for the amount of variation within the subgroups, are there detectable differences between the subgroup averages?" Thus, the *average chart looks for differences between the subgroups*.

The Range Chart asks the question: "Is the variation within subgroups consistent from subgroup to subgroup?" Or alternatively, "Making allowance for the *average* amount of variation within the subgroups, are there detectably different amounts of variation in the different subgroups?" Thus, the *range chart checks for consistency within the subgroups*.

The difference in these two questions will be illustrated with several examples.

#### SHEET THICKNESSES

The thirty inch wide vinyl sheet used for making the skin of padded dashboards was extruded under the control of an automatic process controller. The input device for this automatic process controller was a traditional beta-scanner which measured the thickness of the vinyl. An engineer wanted to study the thickness readings along one track located ten inches from the left edge of the sheet of vinyl, so he collected all of the data for that track and plotted

them on an average and range chart using subgroups of size four.

By using a subgroup of size four he guaranteed that each subgroup would represent about two minutes of operation of the process. In his judgment this allowed for the routine variation in the extrusion process to show up within each subgroup. The average chart in Figure 1 shows that the automatic controller was adjusting the process up and down in cycles lasting about 20 minutes. While the average thickness was 48.5 mils, it might be 49.5 mils for five or six minutes before dropping to 47 mils six minutes later. This variation in thickness affected the way the vinyl would heat and stretch when vacuum-formed into the shape of a dashboard. This variation in thickness was creating scrap at the next step, but on the average the vinyl was the right thickness!

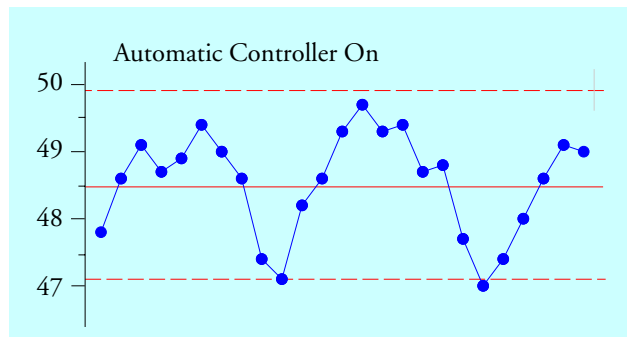
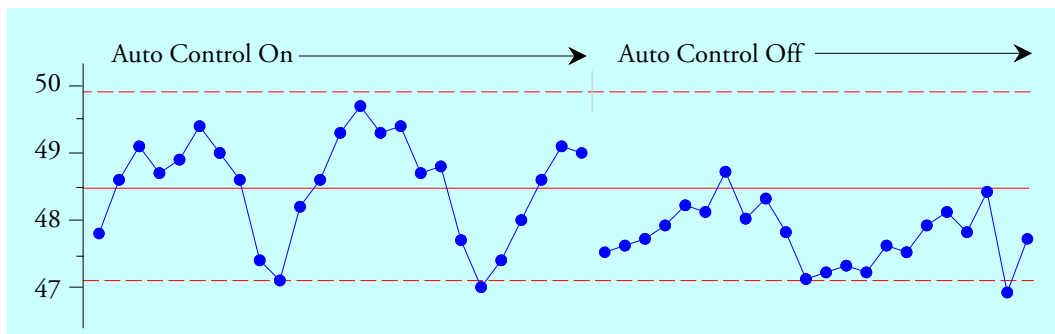


Figure 1: Average Chart for Vinyl Sheet Thicknesses Under Automatic Control

The cycles and the points outside the limits combine to tell us that this automatic process controller is underdamped, not maintaining a good steady-state response, and in need of help. In the words of the engineer who created this chart, it is easy to “recognize a sine-wave” when we see one. Based on the graph in Figure 1, the engineer reached over and turned off the automatic process controller. Over the next 45 minute period he got the values shown on the right in Figure 2.



both the context for the data and how those data are organized into subgroups.

#### TIMES TO MAXIMUM TORQUE

To characterize the curing properties of batches of rubber compound a sample from each batch would be tested in the lab. This test would measure the torque of the rubber sample while it cured. The cure time required to develop maximum torque was the test result. Since each operator produced five batches of rubber each on each shift, the lab decided to use the five daily values for each operator as their subgroups. This resulted in one subgroup per shift, with the within subgroup variation being the variation from batch to batch for each operator, and the between subgroup variation being the variation from operator to operator and from day to day. Since the operators were all producing one product using one rubber mill, we would expect to see a predictable process when we look at the average and range chart. The range chart they obtained is shown in Figure 3.

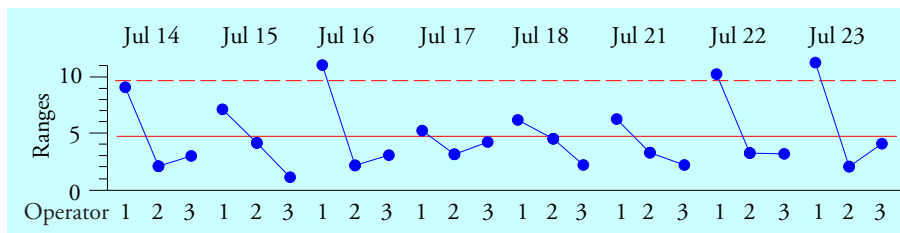


Figure 3 Range Chart for Times to Max Torque

The range chart shows a repeating high-low-low pattern. The batches produced by Operator 1 show more variation than the batches produced by Operators 2 and 3. While Operator 1 was the senior operator, with 30 years experience, he was not getting his batches mixed properly. It turned out that this was because Operator 1 was losing his vision, and could not see well enough to do the manual mixing required.

Once again, the key to interpreting the data is the organization of the data on the chart. The range chart shows the lack of consistency within the subgroups, and the identification of each subgroup with a single operator allows us to make sense of the pattern in Figure 3. It is the organization of the data that determines what questions will be addressed by the average and range chart. **Changes in location that occur between the subgroups will show up on the average chart. Changes in variation that occur within the different subgroups will show up on the range chart.** In each case, it is the variation within the subgroups that defines the yardstick for detecting any differences that occur. Understanding this is the key to the effective analysis of observational data.

In the first example above it was a consistent pattern that suggested the simple experiment of turning off the automatic process controller. In the second example it was a persistent pattern that fit in with the structure of the data that led to the discovery of the blind operator. In both cases it was the interpretation of the charts in their context that led to the discoveries. This alertness for patterns which fit in with the way the data are collected and plotted cannot be programmed. It depends on someone taking the time and making the effort to look at, *and to think about*, the charts. This has always been, and always will be, an essential part of the effective

use of process behavior charts.

For some data sets the rational subgrouping will be clear. However, for some data sets there may be several possible ways of subgrouping the data. The next example falls into this category.

### INJECTION MOLDED BALL JOINT SOCKETS

Injection molding is used to make a ball-joint socket, four pieces at a time. At the time these data were collected, this method of fabrication represented a shift in both material and technology. Therefore, a process certification run was required prior to going into full production. Dave, the supervisor, decided to use process behavior charts to evaluate this process certification run.

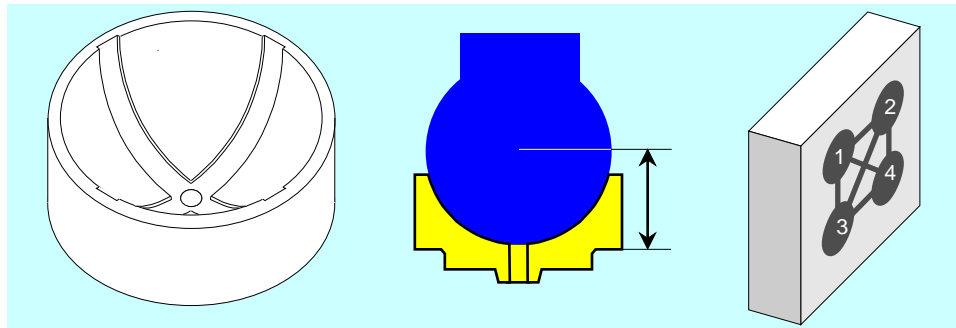


Figure 4: The Ball Joint Socket, the Thickness Dimension, and the Four Cavity Mold

Since there was only one mold, the process certification run involved only one press and only one operator. The data consisted of the effective thickness of a socket, measured in hundredths of a millimeter. Since one side of the socket was convex, a special gauge had to be designed and built to measure this thickness. Because of the way this gauge was built, the measurements represent the thickness in excess of 12.00 millimeters. Four times each day Dave would go to the press and gather up the parts produced by five consecutive cycles of the press. Since each cycle produced four parts, (one from each cavity) he had 20 parts to measure every two hours. Being careful, Dave kept track of the cycle and the cavity from which each part came.

Hour 1	Consecutive Cycles					2	Consecutive Cycles					3	Consecutive Cycles				
	A	B	C	D	E		A	B	C	D	E		A	B	C	D	E
<i>Cavity I</i>	15	16	17	16	18	<i>I</i>	13	18	15	15	15	<i>I</i>	14	14	18	14	14
<i>Cavity II</i>	10	13	11	10	10	<i>II</i>	9	10	11	8	9	<i>II</i>	9	10	10	12	9
<i>Cavity III</i>	7	8	10	7	10	<i>III</i>	7	11	10	10	9	<i>III</i>	8	10	9	10	8
<i>Cavity IV</i>	8	9	10	10	10	<i>IV</i>	10	13	13	10	9	<i>IV</i>	8	9	10	8	9

Figure 5: Structure of Hourly Socket Thickness Data

There are three identifiable sources of variation within these data. There is the Hour-to-Hour variation that is represented by the different blocks of 20 parts. There is the Cycle-to-Cycle variation that is represented by the different columns in Figure 5. And there is the Cavity-to-Cavity variation that is represented by the different rows in Figure 5. We shall look at different ways of organizing these data for an average and range chart and the effect of each

organization upon the interpretation of the charts. For the certification run Dave collected data for six days. In the interest of brevity, we shall only use the data from the first two days.

The complete data set and the first organization is shown in Figure 6. There each column of four values is used to define a subgroup, so that our 160 values are organized into 40 subgroups of size four. The data from different hours (1, 2, 3, etc.) are in different subgroups. When you change hours you are changing subgroups. Therefore, in this first organization, the hour-to-hour differences (and also the day-to-day differences) can be said to show up *between* the subgroups. Here the average chart will ask the following question:

**Question 1. Are there detectable differences between hours or between days?**

In Figure 6 the data from different cycles (*A, B, C, D, E*) are found in different subgroups. When you change cycles you are changing subgroups. Therefore, the cycle-to-cycle differences can be said to show up *between* the subgroups in this first organization of these data. Here the average chart will also ask the following question:

**Question 2. Are there detectable differences between cycles?**

In Figure 6 the data from different cavities (*I, II, III, IV*) are found in the same subgroup. When you change cavities you do not have to change subgroups. Therefore, the cavity-to-cavity differences can be said to show up *within* the subgroups in this first organization of these data. So, here the range chart will ask the following question:

**Question 3. Are the cavity-to-cavity differences consistent?**

1:	A	B	C	D	E	2:	A	B	C	D	E	3:	A	B	C	D	E
I	15	16	17	16	18	I	13	18	15	15	15	I	14	14	18	14	14
II	10	13	11	10	10	II	9	10	11	8	9	II	9	10	10	12	9
III	7	8	10	7	10	III	7	11	10	10	9	III	8	10	9	10	8
IV	8	9	10	10	10	IV	10	13	13	10	9	IV	8	9	10	8	9
$\bar{X}$	10	11.5	12	10.75	12	$\bar{X}$	9.75	13	12.25	10.75	10.5	$\bar{X}$	9.75	10.75	11.75	11	10
R	8	8	7	9	8	R	6	8	5	7	6	R	6	5	9	6	6
4:	A	B	C	D	E	5:	A	B	C	D	E	6:	A	B	C	D	E
I	14	15	15	15	14	I	12	13	13	12	13	I	14	15	17	14	13
II	10	10	11	11	10	II	8	7	9	7	6	II	7	11	11	8	7
III	7	9	12	10	10	III	5	6	8	5	4	III	6	4	7	6	5
IV	11	12	11	10	13	IV	4	4	5	3	4	IV	4	7	6	5	4
$\bar{X}$	10.5	11.5	12.25	11.5	11.75	$\bar{X}$	7.25	7.5	8.25	6.75	6.75	$\bar{X}$	7.75	9.25	10.25	8.25	7.25
R	7	6	4	5	4	R	8	9	8	9	9	R	10	11	11	9	9
7:	A	B	C	D	E	8:	A	B	C	D	E						
I	12	12	13	13	11	I	13	15	16	14	13						
II	6	6	6	6	7	II	9	8	7	7	8						
III	4	5	4	4	6	III	10	8	6	7	5						
IV	4	4	5	4	5	IV	6	6	8	6	5						
$\bar{X}$	6.5	6.75	7	6.75	7.25	$\bar{X}$	9.5	9.25	9.25	8.5	7.75						
R	8	8	9	9	6	R	7	9	10	8	8						

Figure 6: First Organization of the Socket Data

The grand average is 9.54, the average range is 7.63, resulting in the limits shown in Figure 7. By leaving selected line segments out we make the graph easier to read by giving our eyes an anchor to see each hour separately. While no points fall outside the limits, there is a clear signal on the average chart. With 20 out of 20 averages above the central line followed by 19 out of 20 below the central line there is a real difference between days one and two. There may also be a day-to-day difference showing up on the range chart. So we answer question 1 with a definite yes, we answer question 2 with a no, and we answer question 3 with a probable no.

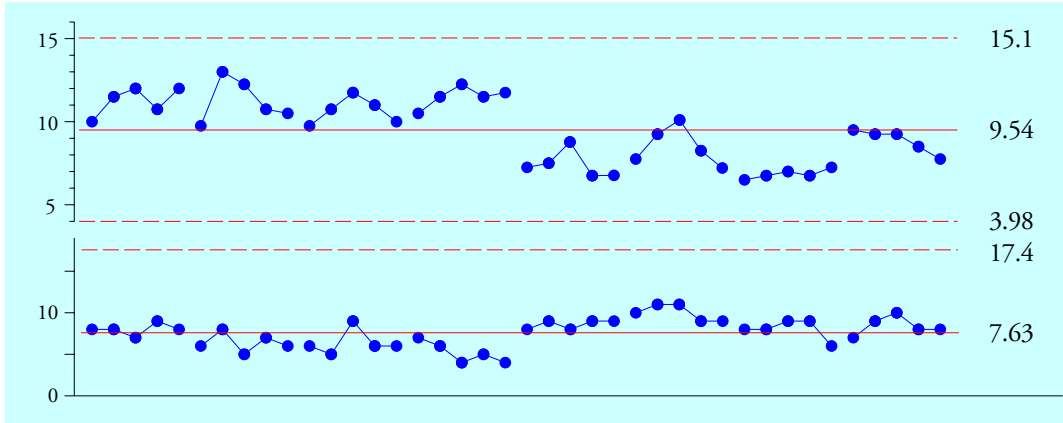


Figure 7: Average and Range Chart for First Organization of the Socket Data

THE SECOND ORGANIZATION OF THE SOCKET DATA

1:	A	B	C	D	E	$\bar{X}$	R	2:	A	B	C	D	E	$\bar{X}$	R	3:	A	B	C	D	E	$\bar{X}$	R
I	15	16	17	16	18	16.4	3	I	13	18	15	15	15	15.2	5	I	14	14	18	14	14	14.8	4
II	10	13	11	10	10	10.8	3	II	9	10	11	8	9	9.4	3	II	9	10	10	12	9	10	3
III	7	8	10	7	10	8.4	3	III	7	11	10	10	9	9.4	4	III	8	10	9	10	8	9	2
IV	8	9	10	10	10	9.4	2	IV	10	13	13	10	9	11	4	IV	8	9	10	8	9	8.8	2
4:	A	B	C	D	E	$\bar{X}$	R	5:	A	B	C	D	E	$\bar{X}$	R	6:	A	B	C	D	E	$\bar{X}$	R
I	14	15	15	15	14	14.6	1	I	12	13	13	12	13	12.6	1	I	14	15	17	14	13	14.6	4
II	10	10	11	11	10	10.4	1	II	8	7	9	7	6	7.4	3	II	7	11	11	8	7	8.8	4
III	7	9	12	10	10	9.6	5	III	5	6	8	5	4	5.6	4	III	6	4	7	6	5	5.6	3
IV	11	12	11	10	13	11.4	3	IV	4	4	5	3	4	4	2	IV	4	7	6	5	4	5.2	3
7:	A	B	C	D	E	$\bar{X}$	R	8:	A	B	C	D	E	$\bar{X}$	R								
I	12	12	13	13	11	12.2	2	I	13	15	16	14	13	14.2	3								
II	6	6	6	6	7	6.2	1	II	9	8	7	7	8	7.8	2								
III	4	5	4	4	6	4.6	2	III	10	8	6	7	5	7.2	5								
IV	4	4	5	4	5	4.4	1	IV	6	6	8	6	5	6.2	3								

Figure 8: Second Organization of the Socket Data

The second organization of these data is shown in Figure 8. There each row of five values is used to define a subgroup so that we end up with 32 subgroups of size five. Here the data from different hours (1, 2, 3, etc.) are in different subgroups. When you change hours you are changing subgroups. Therefore, in this second organization, the hour-to-hour (and day-to-day) differences can be said to show up *between* the subgroups. Here the average chart will ask the following question.

**Question 4. Are there detectable differences between hours or between days?**

In Figure 8 the data from different cycles (A, B, C, D, E) are found in the same subgroup. When you change cycles you do not have to change subgroups. Therefore, the cycle-to-cycle differences can be said to show up *within* the subgroups in this second organization of these data. Here the range chart will ask the following question.

**Question 5. Are the cycle to cycle differences consistent?**

In Figure 8 the data from different cavities (I, II, III, IV) are found in different subgroups. When you change cavities you are changing subgroups. Therefore, the cavity-to-cavity differences can be said to show up *between* the subgroups in this second organization of these data. Here the average chart also asks the following question.

**Question 6. Are there detectable differences between cavities?**

The grand average is 9.54, the average range is 2.84, resulting in the limits shown in Figure 9. With 20 of our 32 averages outside the limits we have plenty of signals to interpret. There are detectable differences between the two days and there are detectable differences between the four cavities. Moreover, the cycle-to-cycle variation appears to be consistent from subgroup to subgroup.

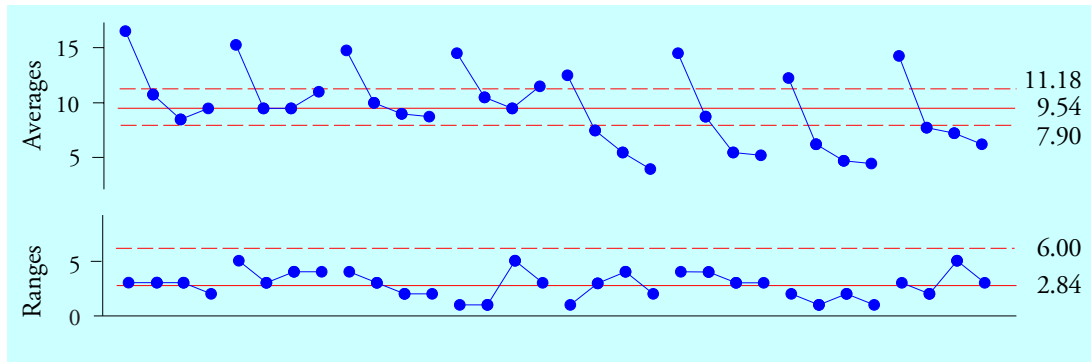


Figure 9: Average and Range Chart for Second Organization of Socket Data

Both of the organizations above are technically correct, but they are not the same in practice because they do not ask the same questions of the data. To understand this difference consider Question 3 and Question 6.

The first organization resulted in Question 3 which asked “Are the cavity-to-cavity differences consistent?” The range chart in Figure 7 answered this question in the affirmative. The cavity-to-cavity differences are consistent.

The second organization resulted in Question 6 which asked “Are there detectable differences between cavities?” The average chart in Figure 9 answered this question in the affirmative. There are detectable differences between the four cavities. Cavity I is making consistently thicker parts than the other cavities.

Until you understand the difference between Question 3 and Question 6, and until you understand how to use this difference to obtain answers to your questions of interest, you will not understand rational subgrouping. This is a skill that requires practice and thought. You can practice by answering the questions in the next section.

### THE THIRD ORGANIZATION OF THE SOCKET DATA

Dave, the supervisor, did not use either of the preceding organizations. Instead he used the organization in Figure 10 for his certification run. Once again we use each row of five values as a subgroup of size five, so the subgroups are the same as in the second organization, but now we organize them differently. Rather than one chart with 32 subgroups, we will have a separate chart for each cavity.

In Figure 10, while holding cavity and cycle fixed, are you changing subgroups as you go from hour to hour?

So are the hour-to-hour differences to be found within subgroups or between subgroups?

So where will the hour-to-hour differences show up: the range chart or the average chart?

In Figure 10, while holding hour and cavity fixed, are you changing subgroups as you go from cycle to cycle?

So are the cycle-to-cycle differences to be found within subgroups or between subgroups?

So where will the cycle-to-cycle differences show up: the range chart or the average chart?

In Figure 10, while holding hour and cycle fixed, are you changing subgroups as you go from cavity to cavity? Are you changing charts?

So where are the cavity-to-cavity differences to be found?

So where will the cavity-to-cavity differences show up?

**Cavity I:**

Hour	A	B	C	D	E	$\bar{X}$	R	Hour	A	B	C	D	E	$\bar{X}$	R	Hour	A	B	C	D	E	$\bar{X}$	R
1:	15	16	17	16	18	16.4	3	2:	13	18	15	15	15	15.2	5	3:	14	14	18	14	14	14.8	4
4:	14	15	15	15	14	14.6	1	5:	12	13	13	12	13	12.6	1	6:	14	15	17	14	13	14.6	4
7:	12	12	13	13	11	12.2	2	8:	13	15	16	14	13	14.2	3								

**Cavity II:**

Hour	A	B	C	D	E	$\bar{X}$	R	Hour	A	B	C	D	E	$\bar{X}$	R	Hour	A	B	C	D	E	$\bar{X}$	R
1:	10	13	11	10	10	10.8	3	2:	9	10	11	8	9	9.4	3	3:	9	10	10	12	9	10	3
4:	10	10	11	11	10	10.4	1	5:	8	7	9	7	6	7.4	3	6:	7	11	11	8	7	8.8	4
7:	6	6	6	6	7	6.2	1	8:	9	8	7	7	8	7.8	2								

**Cavity III:**

Hour	A	B	C	D	E	$\bar{X}$	R	Hour	A	B	C	D	E	$\bar{X}$	R	Hour	A	B	C	D	E	$\bar{X}$	R
1:	7	8	10	7	10	8.4	3	2:	7	11	10	10	9	9.4	4	3:	8	10	9	10	8	9	2
4:	7	9	12	10	10	9.6	5	5:	5	6	8	5	4	5.6	4	6:	6	4	7	6	5	5.6	3
7:	4	5	4	4	6	4.6	2	8:	10	8	6	7	5	7.2	5	9:	9	9	6	8	10	8.4	4

**Cavity IV:**

Hour	A	B	C	D	E	$\bar{X}$	R	Hour	A	B	C	D	E	$\bar{X}$	R	Hour	A	B	C	D	E	$\bar{X}$	R
1:	8	9	10	10	10	9.4	2	2:	10	13	13	10	9	11	4	3:	8	9	10	8	9	8.8	2
4:	11	12	11	10	13	11.4	3	5:	4	4	5	3	4	4	2	6:	4	7	6	5	4	5.2	3
7:	4	4	5	4	5	4.4	1	8:	6	6	8	6	5	6.2	3								

Figure 10: The Third Organization of the Socket Data

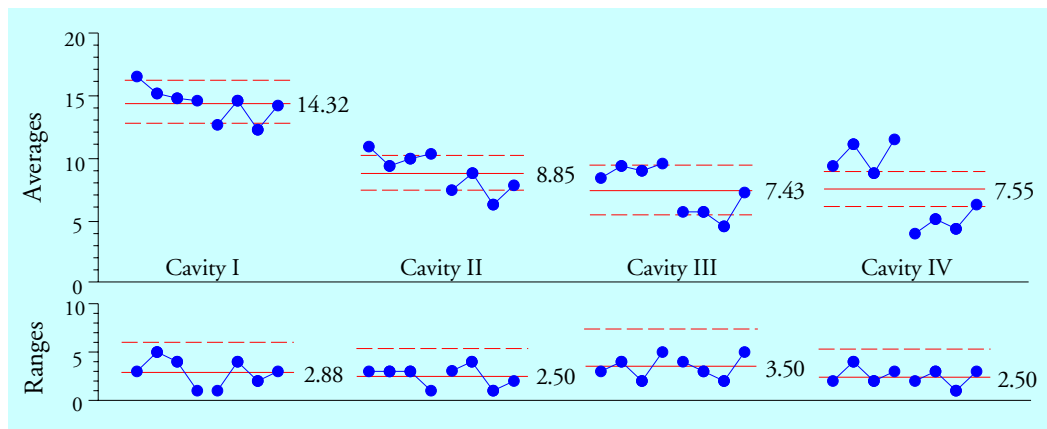


Figure 11: Average and Range Chart for the Third Organization of the Socket Data

By plotting all four charts on the same vertical scale we show the differences between cavities. Clearly Cavity I is making thicker parts, and Cavity II is slightly higher than Cavities III and IV. Based on these charts Dave knew he needed to make an adjustment to the mold. Since Cavities III and IV were fairly well centered in the specs he asked the tool room to place shims



behind Cavities I and II.

What source of variation shows up on the range charts? Hours?, Cycles?, Cavities?

What source of variation shows up on the average charts? Hours?, Cycles?, Cavities?

So what do the points outside the limits on the average charts above represent?

If you had trouble with the previous questions you may need to read this article again.

#### SUMMARY FOR SOCKET DATA

While all three organizations of these data are technically correct, they are not practically equivalent. The different organizations ask different questions of the data, and they make different assumptions regarding the data.

The first organization checks for consistency cavity to cavity, and looks for differences between hours and between cycles.

The second organization checks for consistency cycle to cycle, and looks for differences between hours and between cavities. Why is this organization more sensitive than the first one?

The third organization also checks for consistency cycle to cycle, and looks for differences between hours and between cavities, but by placing the cavities on separate charts it makes it easier to spot the hour-to-hour and day-to-day differences in this process. Of the three organizations, the third is the best for these data.

#### RATIONAL SUBGROUPING

The key to making an average and range chart answer your questions is understanding how the two portions of the chart ask different questions. You control the questions by means of what sources of variation you place within the subgroups and what sources of variation you place between the subgroups. Things that might be different belong in different subgroups. Things that are likely to be the same belong in the same subgroup.

**When we place two measurements together in the same subgroup we are making a judgment that those two values were obtained under essentially the same conditions.** It is this element of judgment that makes your subgrouping rational. Without such judgment, your subgrouping may well be irrational.

**Never knowingly subgroup unlike things together.** Each subgroup must be logically homogeneous. When you combine apples and oranges and bananas together you may get a good fruit salad, but you will have lousy subgroups. Fortunately, the range chart can warn you when you have systematically combined unlike things together within the subgroups. Figure 12 shows the range chart from Figure 7. There we had all four cavities within each subgroup.

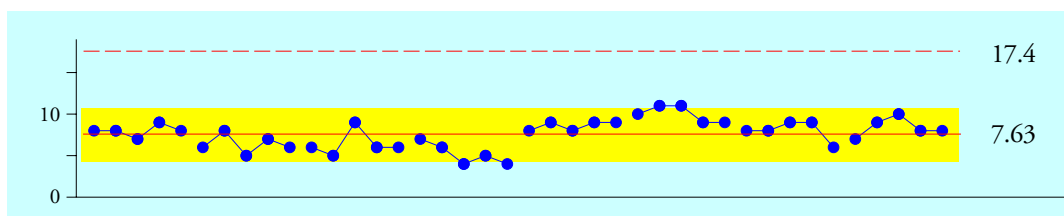


Figure 12: Range Chart from First Organization of Socket Data

The highlighted band in Figure 12 is a one-sigma band. We expect anywhere from 60% to 75% of the range values to fall within this band. Here we get 36 out of 40, which is 90% within one-sigma of the central line. These ranges are hugging the central line, which is a characteristic of having subgroups that are stratified by having unlike things grouped together. The usual guideline for hugging the central line is having 15 successive ranges within one-sigma of the central line of the range chart. If you find this, check for possible stratification within the subgroups. To understand how stratification within the subgroups affects the average chart compare the average chart limits of Figure 7 (basically 4 to 15) with those of Figure 9 (basically 8 to 11).

**Minimize the variation within the subgroups.** It is the variation within the subgroups that defines the background noise level. Any signals will have to be found against this background of noise. By minimizing the variation within the subgroups you will maximize the sensitivity of the process behavior chart.

**Maximize the opportunity for variation between the subgroups.** This requires thought about what types of potential signals might occur within your data stream. If you wish to compare two things, they have to be placed in different subgroups. If there is any possibility that two things might be different, they belong in different subgroups.

**Do not bury signals within the subgroups.** Subgrouping is effective only to the extent that the subgroups remain homogeneous. In many areas of statistics, where estimation of parameters is the objective, larger amounts of data are preferred. This is not the case with average and range charts. Increasing the subgroup size is a good way to destroy the homogeneity of the subgroups. Since the computations explicitly assume internal homogeneity for the subgroups, the logical homogeneity of the subgroups is vastly more important than is the subgroup size.

**Respect the context for your data.** The context defines the structure within your data and is the key to discovering the assignable causes when your process changes. Even the order of the subgroups can be important. This is why we commonly use the time order for the chart. However, you can use other orders when they are meaningful in the context of the data.

Next month I will look at the related topic of Rational Sampling.

## FINAL EXAMINATION

What was the implicit assumption of Figure 7 that turned out to be erroneous?