What is the Precision to Tolerance Ratio?

And does it define a good measurement system?

Donald J. Wheeler

In a class last month I was asked to explain a number that occurs in some measurement system evaluations and which is known as the precision to tolerance ratio (*P/T* ratio). As I will show in this column it turns out to be related to the capability ratio.

We will need some notation in what follows. So, let the product measurements be denoted by X. These product measurements may be thought of as consisting of two components. The value of the item being measured may be denoted by Y, while the error of the measurement may be denoted by E. Thus, $X = Y + E$. If we think about these quantities as variables, then the variation in the product measurements, *Var(X)*, can be thought of as:

$$Var(X) = Var(Y) + Var(E)$$

where *Var(Y)* is the variation in the stream of product values and *Var(E)* is the variation in the stream of measurement errors.

The central purpose of a measurement system evaluation is to obtain an estimate of *Var(E).* Of course, once we have such an estimate the question of how to use it will arise. For more than 40 years it has been common to compare the standard deviation of measurement error with the specified tolerance for a particular product in an attempt to determine the relative utility of the measurement system for a particular product. If we denote the standard deviation of the measurement system by *SD(E)*, and denote the specified tolerance by [*USL – LSL*], then the *P/T* ratio is commonly computed as:

$$P/T = \frac{6\ SD(E)}{USL - LSL}$$

Prior the 1990s the number 5.15 was used rather than the number 6.00. In either case, gauge R&R studies commonly define *P/T* values that are less than 0.10 to be good, while *P/T* ratios in excess of 0.30 are said to be inadequate. To make sense of the *P/T* ratio we will need to establish how it is related to the interclass correlation coefficient and the capability ratio.

In my December 2010 column I showed how the traditional and theoretically correct measure of relative utility is the intraclass correlation coefficient:

$$\textit{Intraclass Correlation Coefficient} = ICC = \frac{Var(Y)}{Var(X)} = 1 - \frac{Var(E)}{Var(X)}$$

This ratio defines that proportion of the variation in the product measurements that can be attributed to the product stream, and it is also the complement of that proportion of the variation in the product measurements that can be attributed to the measurement system. From the last expression above we can establish that:

$$Var(X) = \frac{Var(E)}{1 - ICC}$$

Taking the square root of each side we get:

$$SD(X) = \frac{SD(E)}{\sqrt{1 - ICC}}$$

Next, we need to recall that the capability ratio is defined as the specified tolerance divided by six times the standard deviation of the product measurements, *X*.

$$\text{Capability Ratio} \;=\; C_p \;=\; \frac{USL - LSL}{6\,SD(X)}$$

Using the relationship between *SD(X)* and *SD(E)* given above we can rewrite the capability ratio in terms of measurement error and the intraclass correlation coefficient as:

$$C_p \;=\; \frac{USL - LSL}{6\,SD(E)}\sqrt{1 - ICC} \;=\; Lambda\,\sqrt{1 - ICC}$$

For a given measurement system and a given set of product specifications the value for *Lambda* will remain fixed and both the capability ratio and the interclass correlation coefficient will depend upon *Var(Y)*. Specifically, as improvements are made to the production process, *Var(Y)* will decrease, the capability ratio will increase, and the intraclass correlation coefficient will get smaller. In the limit, as *ICC* goes to zero, the capability ratio will increase towards an upper bound defined by the value for *Lambda*. However, since *Var(Y)* cannot go to zero, neither can *ICC*, and *Lambda* remains a forever unreachable upper bound for the capability ratio.

Inspection of the expression above will show that *Lambda* is simply the inverse of the *P/T* ratio. This means that the *P/T* ratio is the inverse of the *unobtainable* upper bound for the capability ratio. It is the inverse of an impossible value. The following expression summarizes the relationship between the capability ratio, the *P/T* ratio, and the intraclass correlation coefficient:

$$C_p \;=\; \frac{\sqrt{1 - ICC}}{P/T}$$

Now that we know how the *P/T* ratio is related to other quantities, we turn to the question of whether we can use it to define when a measurement system is adequate. The general guideline found in various gauge R&R studies is that good measurement systems will have a *P/T* ratio that is less than 0.10, while merely adequate measurement systems will have a *P/T* ratio that is less than 0.30.

Asking for a *P/T* ratio to be smaller than 0.30 is like asking for an upper bound of 3.33 or greater for your capability ratios. While having the ability to compute such capability ratios is good, we can, and often do, work with measurement systems that cannot deliver such rarefied capability ratios. Even if it made sense to define the relative utility of a measurement system in terms of an upper bound on the capability ratios, there is hardly anything special about a capability ratio of 3.33 except that it is larger than we typically encounter.

The arbitrary nature of the guideline above may be seen in the following manner. Those gauge R&R studies that use the above guideline for the *P/T* ratio, also tend to include a second guideline that requires the gauge repeatability and reproducibility (*GRR*) to be less than 0.30 for an adequate measurement system. The *GRR* value is defined by the ratio:

$$GRR \;=\; \frac{SD(E)}{SD(X)}$$

Inspection of the earlier expression for the interclass correlation coefficient will show that:

$$ICC \;=\; 1 - GRR^{2}$$

So that when *GRR* is 0.3 we will have an *ICC* of 0.91. Using the guidelines that define an adequate measurement system as one with a *P/T* ratio of 0.3 and a *GRR* value of 0.30, we find from the formula relating capability, *ICC*, and *P/T* that you will automatically end up with a capability ratio of 1.00. Once you get to this point you are in a bind. Any reduction in *Var(Y)* that would increase the capability ratio (a desirable outcome) will simultaneously increase the *GRR* value into the range where the guidelines will characterize the measurement system as inadequate (an undesirable outcome). This use of guidelines for multiple characteristics effectively over-constrains the definition of an acceptable measurement system.

Hence, the use of the *P/T* ratio to characterize the relative utility of a measurement system for a particular product is both inappropriate and ineffective.

The correct, sound, and appropriate measure of relative utility is the interclass correlation coefficient. In my December 2010 column I showed how measurement systems having intraclass correlations in excess of 0.20 (*GRR* values as large as 0.89) are able to track process changes. This effectively shows that the guidelines commonly used in gauge R&R studies are excessively conservative and arbitrary. Learn how to use the right measure, and you will no longer have to depend upon arbitrary guidelines to interpret the inverse of impossible values.