ANOX: The Analysis of Individual Values

A new test for homogeneity

Donald J. Wheeler and James Beagle III

Sometimes we use a chart for individual values and a moving range (an XmR chart) to assess the homogeneity of a finite data set. Since this is an "off-label" use for the XmR chart, we first consider the drawbacks associated with using a sequential technique as a one-time test, and then present an adaptation of the *X* chart (the Analysis of Individual Values or ANOX) that functions like other one-time statistical tests.

THE XmR CHART

Like all process behavior charts, the XmR chart was designed for the *sequential* analysis of a *continuing stream* of observational data. Here the data will generally represent one condition, and the purpose of the chart is to identify unplanned changes in the underlying process. After the baseline period, where we compute the limits, we extend the limits forward and continue to add data to the chart. *Each time we add an additional point to the chart we are performing an act of analysis*. Each of these analyses asks if the current value is consistent with the baseline period. And, as with all sequential procedures, we want to perform each of these acts of analyses in a conservative manner in order to reduce the overall risk of a false alarm.

However, we do have to get started, and we do this with a baseline period. This choice of a baseline period is a matter of judgment. It has to make sense in the context of the application. Moreover, since future values will be compared to the baseline period, the choice of the baseline period will define the questions framed by the process behavior chart.

How many data are needed for the baseline period? While a rational baseline may end up using as few as four or five points, we prefer to have 17 to 30 points in our baseline when the context allows us to do so. On the high side, we will seldom need more than 50 points for a rational baseline on an XmR chart.

Once we have selected our baseline period and computed our three-sigma limits according to the usual formula [*Average* ± 2.66 (*Average Moving Range*)], we are ready to begin the sequential analysis portion of the XmR chart. As we do this the inherently conservative nature of the three-sigma limits minimizes the risk of a false alarm *for each individual act of analysis* regardless of the shape of the histogram.

As an example consider some data from the classroom. A box containing 4800 beads of different colors is sampled, with replacement, using a paddle with 50 holes. The paddle is dipped into the box, pulled out, and the number of yellow beads in the sample of 50 beads is recorded. After 20 such draws we have 20 counts, and these counts are used to create a baseline for future drawings from this bead box. The baseline portion of the XmR chart is shown on the left side of Figure 1. It has an average of 5.15 and an average moving range of 3.26.

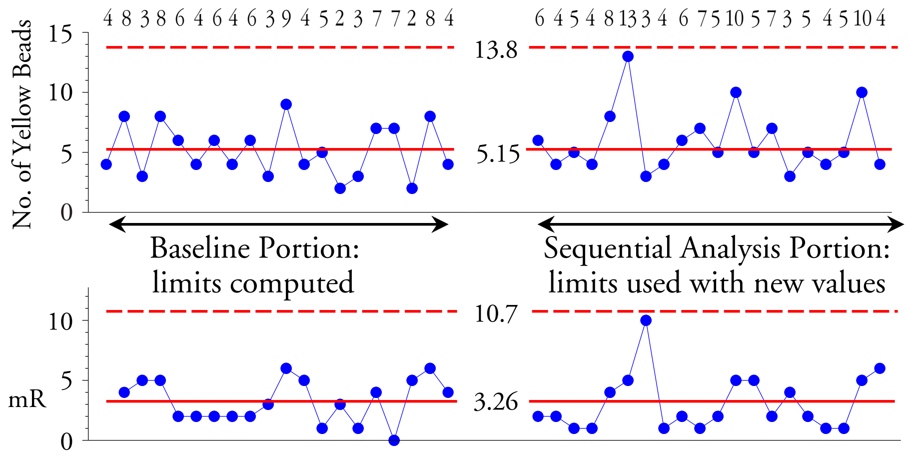

Figure 1: XmR Chart for the Number of Yellow Beads Out of 50 Beads

Next, as each additional drawing is made, the count of yellow beads is placed on the process behavior chart. As each point is added to the chart another act of analysis is performed. The question asked is whether the latest point is detectably different from the points in the baseline period? Twenty such sequential acts of analysis are shown on the right side of Figure 1.

The fixed-width, three-sigma limits of the process behavior chart assure us that each of the sequential analyses is done in a conservative manner. This means that whenever we find an individual value outside the limits, we can be confident that that value is different from the values in the baseline period and a change is likely to have occurred in the underlying process. This time-ordered, sequential analysis is the essence of the process behavior chart technique.

ONE-TIME TESTS

When we use a process behavior chart as a one-time test for a finite data set we are no longer characterizing the behavior a continuing process. We are simply using the front-end, baseline portion of the process behavior chart as a test of homogeneity for the data already in hand. This is a fundamental change in usage, with consequences that we will note below. Some examples of where this type of analysis would be in order are:

1. Characterization of a product in an engineering environment: A pilot run of products or assemblies may be produced to assess functional or design parameters. Since this pilot run product may be used to determine specifications for components or for performance parameters, the homogeneity of the physical and functional characteristics of the pilot run will be of interest.

2. Evaluation of new designs in research runs: When a small number of items are evaluated for performance characteristics it is important to know that the items are homogeneous with regard to their design characteristics.

3. Representation: When a sample of items is obtained from a lot, our extrapolation from the sample back to the lot as a whole will depend upon how well the sample represents the lot. This representativeness will depend in part upon the homogeneity of the sample. If the sample is not homogeneous, then it is likely that the lot is also not homogenous. And if the lot is not

homogeneous, no single sample can be said to be truly representative of the whole.

When we use an XmR chart as a one-time test it is usually inappropriate to use run-tests. Run-tests are built on the notion of sequence and patterns within that sequence, but with tests for homogeneity the sequence of the values will often be unknown or arbitrary. For this reason, unless we know and are using the time-order sequence for the data, we should only use points outside the limits as signals of a lack of homogeneity.

ALPHA LEVELS FOR THE XmR CHART

When we use an XmR chart as a one-time test we are simply examining a fixed set of k values for evidence of a lack of homogeneity. With one-time tests it is customary to state the risk of a false alarm. (In the context of a test of homogeneity, a false alarm is equivalent to saying that a homogeneous data set is not homogeneous.)

The false alarm risk for the baseline portion of an XmR chart will be referred to as the "baseline alpha" level to distinguish it from the "alpha" level commonly cited for each individual test for the sequential portion of the XmR chart technique. The relation between these two quantities is given by Bonferroni's inequality. For a baseline containing k values, the baseline alpha will fall in between two values:

$$1 - [\, 1 - \text{alpha} \,]^{k} \;\leq\; \text{baseline alpha} \;\leq\; k \text{ alpha}$$

If we use the traditional, normal-theory alpha value of 0.0027 for the individual tests in the sequential portion of an XmR chart, then a baseline consisting of $k = 19$ points would be said to have a baseline alpha of:

$$1 - [\, 1 - 0.0027 \,]^{19} \;\leq\; \text{baseline alpha} \;\leq\; 19\,(\,0.0027\,)$$

$$0.050 \;\leq\; \text{baseline alpha} \;\leq\; 0.051$$

Thus, when an XmR chart is used as a test for homogeneity for 19 values it can be said to have an overall risk of a false alarm of about 5 percent.

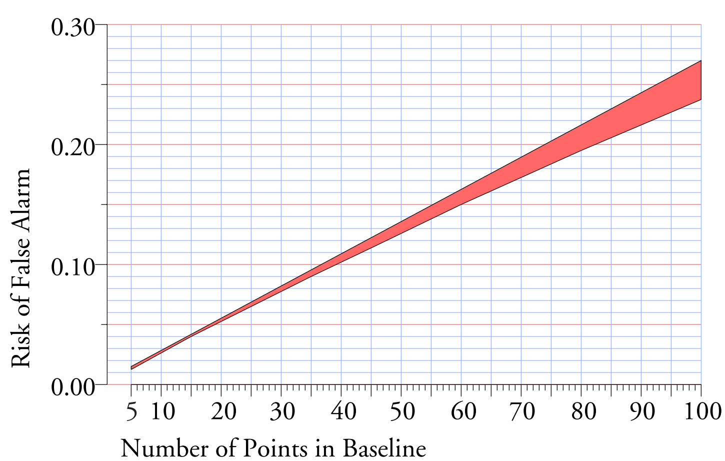

Figure 2: Overall Alpha Levels for XmR Chart Baselines with *k* Values

When we plot the results of the Bonferroni inequality versus the number of values in the baseline period of an XmR chart we get the graph shown in Figure 2. When an XmR chart is used as a one-time test for the homogeneity of *k* values the likelihood of a false alarm will depend upon *k*. This means that, unlike other one-time tests, *we do not get to choose our alpha level when we use an XmR chart as a test for homogeneity.*

Typically with a one-time test we choose our alpha level, perform the test, state our conclusion, and live with the consequences. In some cases it will be more important to avoid false alarms, and in others it will be more important to avoid missed signals. By choosing a small alpha level (say one percent) we can minimize the risk of a false alarm. By choosing a large alpha level (say ten percent) we can reduce the risk of a missed signal. Decision theory shows that for one-time tests the traditional alpha-level of five percent strikes a reasonable balance between both mistakes. Thus, the choice of alpha level is how we show our attitude toward the analysis and fine-tune our one-time test.

So, the drawback to using an XmR chart as a test of homogeneity is the inability to choose our alpha level. Since the increasing risk of a false alarm shown in Figure 2 is a consequence of the fixed-width limits used with the XmR chart, the obvious way to fix this problem is to use variable-width limits. This was the idea behind Ellis Ott's Analysis of Means (ANOM), and it is what we are proposing for the Analysis of Individual Values (ANOX).

THE ANALYSIS OF INDIVIDUAL VALUES (ANOX)

Given a set of *k* individual values and given the *k*–1 successive differences between these values (i.e. *k*–1 moving ranges), the homogeneity of this set of *k* individual values may be examined by computing the limits:

$$Average \pm ANOX_{\alpha} \{ Average\ \ Moving\ Range \}$$

Where the scaling factor $ANOX_{\alpha}$ depends upon the desired alpha-level for the test and the number of individual values being tested. When the *k* values are actually homogeneous these limits will bracket *all* of the individual values 100[1–alpha] percent of the time, and the smallest or largest of the *k* individual values can be expected to exceed these limits only 100[alpha] percent of the time.

If a single individual value falls outside the ANOX limits then you either have a rare event with probability alpha, or you have a non-homogeneous set of individual values. Of course, as more values fall outside the ANOX limits, and as they fall further outside the ANOX limits, the evidence of non-homogeneity becomes stronger.

In applying this test for homogeneity the individual values *must not* be arranged in a ranking (where the values are arranged in either an ascending or descending numerical order). Such orderings undermine the method of successive differences which is the foundation of both the XmR chart and ANOX.

It is always best to use the data in the order in which they naturally occur. If time-order information is available, this ordering is preferred. If the time-order information has been lost, use the order in which the data are presented (as long as it is not a ranking).

ANOX FOR BLAST FURNACE SILICON

As an example, the data in Figure 3 came from a print-out of 63 consecutive measurements of the silicon level in samples of hot metal coming from a blast furnace. The average is 149.9. Here we have no contextual information given along with the values, so we read the values in rows, compute the moving ranges, and find the average moving range to be 70.0.

Figure 11 shows the $ANOX_{.10}$ scaling factor for k = 63 values to be 2.782. Thus we compute 10% ANOX limits of 149.9 ± 2.782 (70.0) = –44.0 to 344.8. Since these data have a boundary value of zero, we report limits of 0 to 344.8, and no values fall outside these limits. The graph in Figure 4 shows these values and their limits.

144	198	88	211	95	181	113
150	190	111	201	90	173	122
180	178	120	155	107	163	108
193	168	138	145	127	158	135
210	137	160	102	142	147	145
225	121	179	83	159	134	158
235	116	200	80	167	128	133
233	85	245	101	178	113	125
228	65	248	106	199	104	112

Figure 3: 63 Measurements of Blast Furnace Silicon

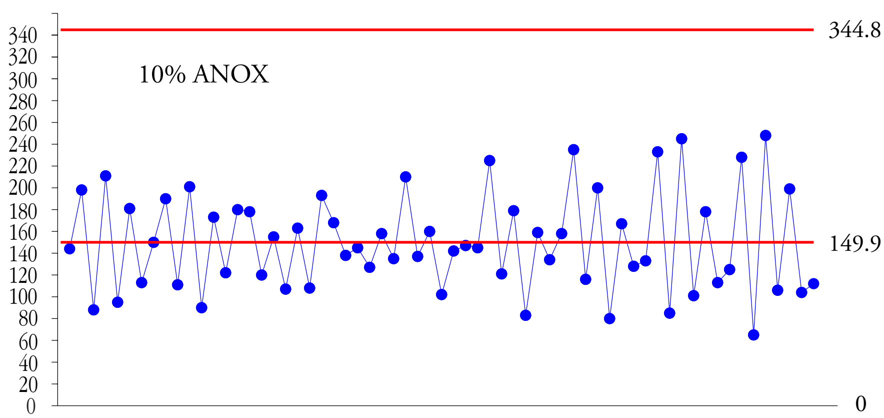

Figure 4: 10% ANOX for Blast Furnace Silicon

Anyone with much experience in looking at process behavior charts will be suspicious of the graph in Figure 4. The running record is a sawtooth and the limits are exceedingly wide compared to the running record. So we might well decide to return to Figure 3 and read the table in columns. When we do this we find that the average moving range drops to 15.9. Our revised 10% ANOX limits become 149.9 ± 2.782 (15.9) = 105.7 to 194.2, and we get the graph in Figure 5. Now we find 21 of the 63 values outside the 10% ANOX limits. Clearly these data are not homogeneous (and the silicon levels in the blast furnace are cycling over time).

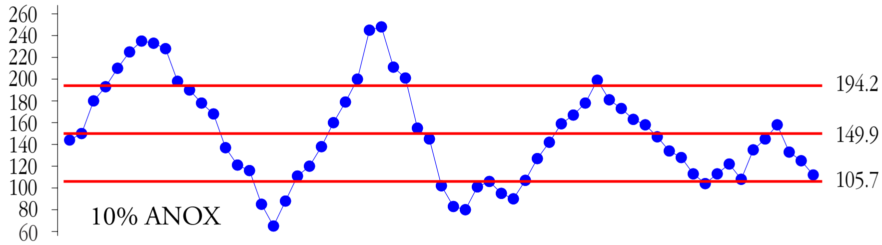

Figure 5: Second 10% ANOX for Blast Furnace Silicon

Here then is an illustration of the point that if *any* arbitrary ordering (other than a ranking) shows evidence of a lack of homogeneity, then the set of values should be considered to be non-homogeneous. (If the data were truly homogeneous, every ordering would tell the same story.)

ASKING THE RIGHT QUESTION

As soon as we determine that a data set is non-homogeneous, all of the traditional statistical techniques that assume homogeneity for our data are off the table. They are no longer relevant. While the statistics will always describe the data set itself, a lack of homogeneity will undermine any attempt to interpret the statistics as representing anything outside the data set. (Our ability to extrapolate from a data set to a broader context will always depend upon the internal homogeneity of the data set.) With a lack of homogeneity the question immediately changes from "What do these data represent?" to "Why are these data not homogeneous?" Until we know the answer to the question of homogeneity we risk asking the wrong question of our data and taking the wrong action as a result.

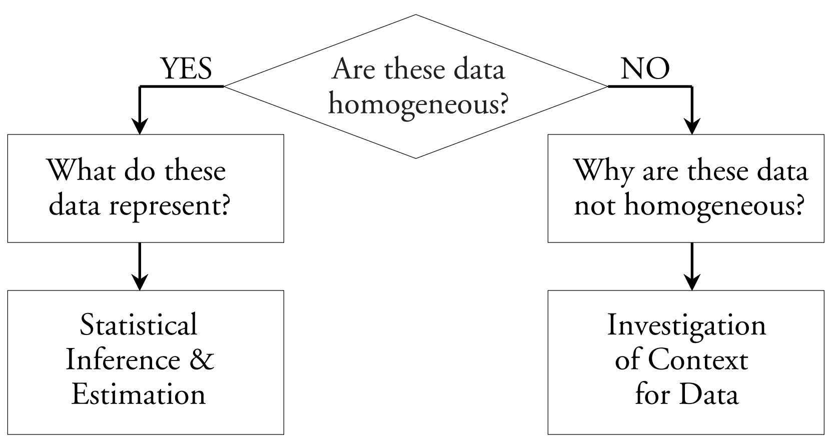

Figure 6: Asking the Right Question Is Essential to Taking the Right Action

TESTING THE SENSORS WITH ANOX

A set of planned experiments involved the use of sensors to detect the levels of specific carbohydrates in solution under various environmental conditions. Since the experimental results would depend upon multiple sensors giving equivalent results, a simple test was run prior to the experiment to evaluate the homogeneity of the collection of sensors available. First the 48 sensors were simultaneously exposed to a zero-load (air) and the current in milliamps passing through each sensor was recorded. Next the sensors were simultaneously exposed to a high-end load using a known solution. Once again the current through each sensor was recorded.

The current readings for the zero-load are shown in Figure 7. Since it is very important that the sensors used in the experiment are all working the same, these values were tested for homogeneity at the 10% alpha level. (It was more important to find and eliminate any bad sensors than to erroneously delete some good sensors.)

The average is 0.8623 mA and the average moving range is 0.3051 mA. With k = 48 the $ANOX_{.10}$ scaling factor is 2.706, giving 10% ANOX limits of 0.037 and 1.688.

ID	1	2	3	4	5	6	7	8
mA	0.85	0.71	0.99	0.99	0.85	0.64	0.71	0.57
ID	9	10	11	12	13	14	15	16
mA	1.07	–0.14	1.07	0.78	1.14	0.99	0.57	0.78
ID	17	18	19	20	21	22	23	24
mA	1.35	0.57	0.92	0.99	0.85	0.57	0.64	1.14
ID	25	26	27	28	29	30	31	32
mA	1.85	0.57	1.07	0.92	0.78	0.78	0.78	0.64
ID	33	34	35	36	37	38	39	40
mA	0.85	1.21	0.78	1.42	1.49	0.85	0.85	0.71
ID	41	42	43	44	45	46	47	48
mA	0.78	0.78	0.71	0.78	0.78	0.99	0.85	0.57

Figure 7: Sensor Currents Under Zero Load Condition

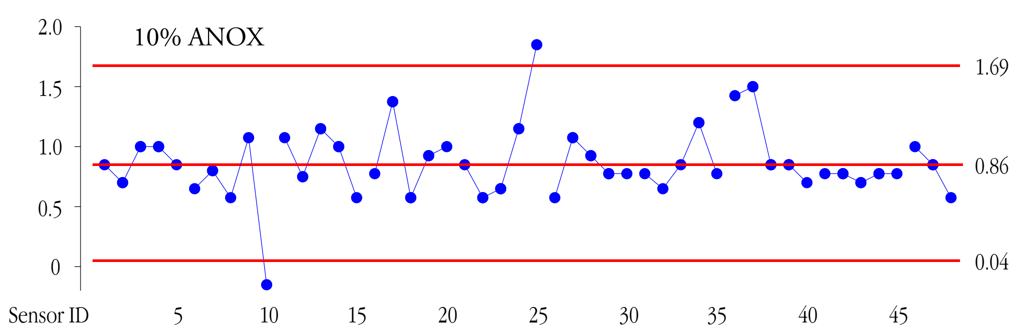

Figure 8: 10% ANOX for Sensor Currents Under Zero Load

Clearly sensors 10 and 25 are different from the rest under the zero-load condition. These two sensors should not be used in the upcoming experiments.

The current readings for the high load condition are shown in Figure 9. The average is 9.896 mA and the average moving range is 0.7689 mA. With k = 48 the $ANOX_{.10}$ scaling factor is 2.706, giving 10% ANOX limits of 7.82 and 11.98.

ID	1	2	3	4	5	6	7	8
mA	10.44	10.24	10.24	9.74	10.04	10.54	10.54	10.04
ID	9	10	11	12	13	14	15	16
mA	10.34	7.06	10.44	9.25	10.54	9.34	8.95	7.56
ID	17	18	19	20	21	22	23	24
mA	9.84	9.15	10.44	9.54	9.44	9.94	9.74	10.34
ID	25	26	27	28	29	30	31	32
mA	10.64	9.15	10.64	9.94	9.64	9.84	10.54	9.94
ID	33	34	35	36	37	38	39	40
mA	10.34	10.24	10.34	10.04	10.84	9.34	10.74	9.54
ID	41	42	43	44	45	46	47	48
mA	10.04	9.05	9.84	10.34	10.04	10.04	10.24	9.94

Figure 9: Sensor Currents Under High Load Condition

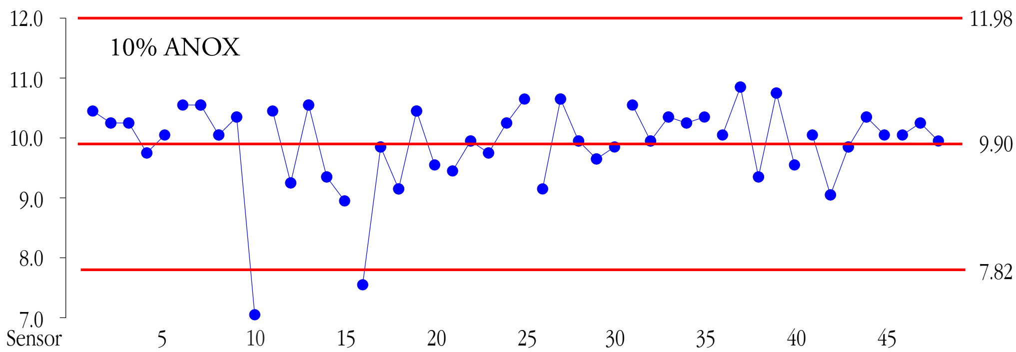

Figure 10: 10% ANOX for Sensor Currents Under High Load

Here we find sensors 10 and 16 to be different from the rest under the high load condition. So, we add sensor 16 to the "do not use" list. By using 10% ANOXs to test for homogeneity we can be confident that the remaining 45 sensors give similar readings under both zero-load and high-load conditions.

THE CHOICE OF ALPHA LEVEL

In most one-time tests we are trying to establish that a signal exists. In a test of homogeneity we are trying to establish that a signal does not exist. This difference will affect the way we use a test for homogeneity.

When we use a 5% ANOX or a 10% ANOX and find no evidence of a lack of homogeneity, then we can be comfortable with the conclusion that the data are probably homogeneous. We will have obtained a fairly strong result.

When we use a 1% ANOX and find evidence of a lack of homogeneity, then we can be comfortable with the conclusion that the data are not homogeneous.

Since the skeptic will assume the data are not homogeneous until proven otherwise, and since the strong conclusion regarding homogeneity requires a larger alpha level, we should be careful about using a 1% ANOX to conclude that the data are homogeneous. In the preceding examples, sensor 10 is outside the 1% ANOX limits in both cases, but sensors 25 and 16 are not.

Would you want to risk using sensors 25 and 16 in your experiments just because they did not exceed the 1% ANOX limits?

In addition, it is important to remember that with ANOX the alpha level is the probability that either the *maximum* or *minimum* of the k points will fall outside the limits. So if we perform a 10% ANOX with k = 100 points, the alpha level of ten percent does not mean that we expect 10 out of a 100 values to fall outside the limits. Instead, it means that there is a ten percent chance that either the smallest or the largest of the 100 values might fall just outside the limits. When a false alarm occurs with ANOX it will tend to show a single point just outside the limit, rather than having a point noticeably outside the limits. The judgment that a data set is not homogeneous can depend upon how far outside the limits a point may fall as well as how many points are outside the limits.

COMPARING ANOX WITH THE XmR CHART

Since the whole of statistical inference is based on the assumption of homogeneity, it is appropriate to begin any analysis with a test of homogeneity.

From the Bonferroni inequality and Figure 2 we may conclude that when we have fewer than 38 data, using an XmR chart as a one-time test will detect fewer signals of non-homogeneity than will a 10% ANOX.

When we have fewer than 19 data, using an XmR chart as a one-time test will detect fewer signals of non-homogeneity than will a 5% ANOX.

However, when we have 8 or more data, using a 1% ANOX will detect fewer signals of non-homogeneity than will be found using an XmR chart as a one-time test.

Thus, ANOX is recommended to avoid the overly large alpha-levels that automatically occur when we use an XmR chart as a one-time test with large data sets. Like virtually all other statistical procedures, ANOX provides you with a one-time test for homogeneity that has a user-selected risk of a false alarm. However, unlike other statistical procedures, ANOX is not built upon the assumption that the data are homogeneous.

APPENDIX ONE: CREATING THE ANOX TABLES

Given a set of k individual values:

$$\left\{ X_1 , \ldots , X_k \right\}$$

denote the largest and smallest of these values by:

$$\begin{aligned} X_{\text{MAX}} &= \text{Maximum of} \left\{ X_1 , \ldots , X_k \right\} \\ X_{\text{MIN}} &= \text{Minimum of} \left\{ X_1 , \ldots , X_k \right\} \end{aligned} \qquad [1]$$

Then the greatest deviation from the Average would be given by:

$$\textit{Greatest Deviation X} = \text{Maximum of} \left[X_{\text{MAX}} - \bar{X} \right] \text{ and } \left[\bar{X} - X_{\text{MIN}} \right] \qquad [2]$$

If we let GDX_{90} denote the 90th percentile of the distribution of the Greatest Deviation Statistic, and if we define

$$GDX_{90} = ANOX_{.10} \;\; [\, Average\ Moving\ Range\,] \qquad [3]$$

then the quantity $ANOX_{.10}$ will be defined by the 90th percentile of the distribution of the ratio:

$$\frac{Greatest\ Deviation\ X}{Average\ Moving\ Range} \qquad [4]$$

where the average moving range statistic in the denominator is the average of the k–1 successive differences between the individual values.

In a similar manner the quantities $ANOX_{.05}$ and $ANOX_{.01}$ will be defined by the 95th and 99th percentiles of the distribution of the ratio in equation [4} above. To estimate these percentiles for different values of k each of the authors independently ran a series of simulation runs. These independent simulations converged on the values given in the tables. (When corresponding results of the independent simulations were tested for detectable differences over 98 percent of the tests found no detectable difference at the five percent level. At the one-percent level there were no detectable differences between the two independent simulations. Thus each author provided non-trivial confirmation for the other's results.)

A simulation run would begin with the generation of 10,000 random samples of size k using a standard normal distribution. For each of these samples the moving ranges were computed and the ratio in equation [4] above was computed. The 99th, 95th, and 90th percentiles of the distribution of the ratio above would be estimated by finding the 100th, 500th, and 1000th values from the ordered set of observed ratios. Thus, for each value of k, a simulation run would yield a single estimate for $ANOX_{.01}$, a single estimate for $ANOX_{.05}$, and a single estimate for $ANOX_{.10}$. By repeating this whole process dozens to hundreds of times for each value of k, and then averaging the resulting estimates, it was possible to obtain estimates for $ANOX_{.01}$, $ANOX_{.05}$, and $ANOX_{.10}$ having very small uncertainties.

Since scaling factors such as these should form a smooth curve when plotted as a function of k, the estimates from the simulation studies were plotted along with their error bars, and the resulting curves were smoothed by adjusting points that seemed high or low relative to adjacent estimates. All but four of the adjusted values remained within the error bars from the simulation study, and these four were only 0.001 or 0.002 outside their error bars. Thus, the smoothed values given in the table form internally consistent sets that are also completely consistent with the simulation studies.

ANOX scaling factors for fewer than eight data are not given because of a quirk of the method of successive differences. For both an XmR chart and ANOX, when k is less than 8, it is only the first or last points that can fall outside the computed limits. This means that ANOX cannot provide a fair test of homogeneity for all the points in a data set until k is eight or greater.

In Figure 11, the $ANOX_{.10}$ values for k less than 170 all have a probable error of 0.001 or less. These values err by 1 unit or less in the third decimal place at least half the time. Hence they are essentially known to three decimal places. The $ANOX_{.10}$ values for k greater than 170 have a probable error of 0.002 or less.

10% ANOX SCALING FACTORS $ANOX_{.10}$
For limits having a 10% chance of exceedence for max or min of k values.
Use when skeptical about homogeneity of the individual values.

k	$ANOX_{.10}$	k	$ANOX_{.10}$	k	$ANOX_{.10}$	k	$ANOX_{.10}$	k	$ANOX_{.10}$	k	$ANOX_{.10}$
8	2.058	41	2.662	74	2.823	107	2.921	140	2.988	173	3.037
9	2.118	42	2.668	75	2.827	108	2.924	141	2.990	174	3.039
10	2.167	43	2.675	76	2.830	109	2.926	142	2.992	175	3.040
11	2.209	44	2.682	77	2.833	110	2.929	143	2.993	176	3.041
12	2.246	45	2.688	78	2.837	111	2.931	144	2.995	177	3.043
13	2.279	46	2.694	79	2.840	112	2.933	145	2.996	178	3.044
14	2.308	47	2.700	80	2.843	113	2.935	146	2.998	179	3.045
15	2.334	48	2.706	81	2.847	114	2.938	147	3.000	180	3.047
16	2.358	49	2.712	82	2.850	115	2.940	148	3.001	181	3.048
17	2.381	50	2.718	83	2.854	116	2.942	149	3.003	182	3.049
18	2.401	51	2.724	84	2.857	117	2.944	150	3.004	183	3.050
19	2.420	52	2.729	85	2.860	118	2.946	151	3.006	184	3.051
20	2.437	53	2.735	86	2.863	119	2.949	152	3.007	185	3.053
21	2.454	54	2.740	87	2.867	120	2.951	153	3.009	186	3.054
22	2.469	55	2.746	88	2.870	121	2.953	154	3.010	187	3.055
23	2.485	56	2.750	89	2.873	122	2.955	155	3.012	188	3.056
24	2.499	57	2.755	90	2.876	123	2.957	156	3.013	189	3.057
25	2.512	58	2.760	91	2.879	124	2.959	157	3.015	190	3.059
26	2.524	59	2.764	92	2.882	125	2.960	158	3.016	191	3.060
27	2.535	60	2.769	93	2.885	126	2.962	159	3.018	192	3.061
28	2.546	61	2.773	94	2.888	127	2.964	160	3.019	193	3.062
29	2.558	62	2.777	95	2.891	128	2.966	161	3.021	194	3.063
30	2.569	63	2.782	96	2.893	129	2.968	162	3.022	195	3.064
31	2.579	64	2.786	97	2.896	130	2.970	163	3.024	196	3.065
32	2.589	65	2.790	98	2.899	131	2.972	164	3.025	197	3.067
33	2.599	66	2.794	99	2.901	132	2.974	165	3.026	198	3.068
34	2.609	67	2.798	100	2.904	133	2.976	166	3.028	199	3.069
35	2.618	68	2.801	101	2.906	134	2.977	167	3.029	200	3.070
36	2.626	69	2.805	102	2.909	135	2.979	168	3.030	240	3.123
37	2.633	70	2.809	103	2.911	136	2.981	169	3.032	300	3.168
38	2.640	71	2.812	104	2.914	137	2.983	170	3.033	360	3.212
39	2.648	72	2.816	105	2.916	138	2.985	171	3.035	420	3.246
40	2.655	73	2.820	106	2.919	139	2.987	172	3.036	480	3.274

Figure 11: 10% ANOX Scaling Factors

In Figure 12, the $ANOX_{.05}$ values for k less than 30 have a probable error of 0.001. The remainder of the $ANOX_{.05}$, values have a probable error of 0.002. This means that all the values in Figure 12 err by 2 units or less in the third decimal place at least half the time. Hence, these values are still effectively known to three decimal places.

5% ANOX SCALING FACTORS $ANOX_{.05}$
For limits having a 5% chance of exceedence for max or min of k values.
Use when neutral about assumption of homogeneity for the individual values.

k	$ANOX_{.05}$	k	$ANOX_{.05}$	k	$ANOX_{.05}$	k	$ANOX_{.05}$	k	$ANOX_{.05}$	k	$ANOX_{.05}$
8	2.279	41	2.860	74	3.010	107	3.099	140	3.160	173	3.210
9	2.343	42	2.866	75	3.013	108	3.101	141	3.162	174	3.211
10	2.389	43	2.873	76	3.017	109	3.103	142	3.164	175	3.213
11	2.432	44	2.879	77	3.020	110	3.105	143	3.165	176	3.214
12	2.468	45	2.885	78	3.023	111	3.107	144	3.167	177	3.215
13	2.498	46	2.890	79	3.027	112	3.109	145	3.168	178	3.217
14	2.526	47	2.896	80	3.030	113	3.111	146	3.170	179	3.218
15	2.550	48	2.901	81	3.033	114	3.113	147	3.172	180	3.219
16	2.575	49	2.906	82	3.036	115	3.116	148	3.173	181	3.220
17	2.595	50	2.911	83	3.039	116	3.118	149	3.175	182	3.222
18	2.614	51	2.916	84	3.042	117	3.120	150	3.177	183	3.223
19	2.632	52	2.922	85	3.044	118	3.122	151	3.178	184	3.224
20	2.648	53	2.927	86	3.047	119	3.124	152	3.180	185	3.225
21	2.665	54	2.932	87	3.051	120	3.126	153	3.181	186	3.226
22	2.681	55	2.937	88	3.054	121	3.128	154	3.183	187	3.228
23	2.695	56	2.941	89	3.057	122	3.129	155	3.184	188	3.229
24	2.708	57	2.946	90	3.060	123	3.131	156	3.186	189	3.230
25	2.720	58	2.950	91	3.062	124	3.133	157	3.187	190	3.231
26	2.730	59	2.954	92	3.065	125	3.135	158	3.189	191	3.232
27	2.741	60	2.959	93	3.067	126	3.137	159	3.190	192	3.233
28	2.751	61	2.963	94	3.070	127	3.138	160	3.192	193	3.235
29	2.762	62	2.966	95	3.072	128	3.140	161	3.193	194	3.236
30	2.772	63	2.970	96	3.075	129	3.142	162	3.195	195	3.237
31	2.782	64	2.974	97	3.077	130	3.144	163	3.196	196	3.238
32	2.791	65	2.978	98	3.080	131	3.145	164	3.197	197	3.239
33	2.801	66	2.981	99	3.082	132	3.147	165	3.199	198	3.240
34	2.810	67	2.985	100	3.085	133	3.149	166	3.200	199	3.241
35	2.820	68	2.989	101	3.087	134	3.150	167	3.202	200	3.242
36	2.826	69	2.993	102	3.089	135	3.152	168	3.203	240	3.289
37	2.833	70	2.997	103	3.091	136	3.154	169	3.204	300	3.329
38	2.840	71	3.000	104	3.093	137	3.155	170	3.206	360	3.365
39	2.847	72	3.004	105	3.095	138	3.157	171	3.207	420	3.396
40	2.854	73	3.007	106	3.097	139	3.159	172	3.209	480	3.421

Figure 12: 5% ANOX Scaling Factors

In Figure 13, the $ANOX_{.01}$, values for k less than 44 have a probable error of 0.002. For k greater than 44 they have a probable error of 0.003 or 0.004. So, while these values have some softness in the third decimal place, this softness is still smaller than the uncertainty introduced by rounding these values off to two decimal places. Therefore, these values are listed with three decimal places, even though the third decimal places are somewhat uncertain.

1% ANOX SCALING FACTORS $ANOX_{.01}$
For limits having a 1% chance of exceedence for max or min of k values.
Use only when gullible about assumption of homogeneity for individual values.

k	$ANOX_{.01}$	k	$ANOX_{.01}$	k	$ANOX_{.01}$	k	$ANOX_{.01}$	k	$ANOX_{.01}$	k	$ANOX_{.01}$
8	2.827	41	3.279	74	3.400	107	3.468	140	3.512	173	3.547
9	2.863	42	3.284	75	3.403	108	3.470	141	3.513	174	3.548
10	2.897	43	3.288	76	3.406	109	3.471	142	3.514	175	3.549
11	2.928	44	3.293	77	3.408	110	3.473	143	3.515	176	3.550
12	2.958	45	3.298	78	3.411	111	3.474	144	3.516	177	3.551
13	2.985	46	3.303	79	3.414	112	3.476	145	3.518	178	3.552
14	3.008	47	3.307	80	3.417	113	3.477	146	3.519	179	3.553
15	3.029	48	3.312	81	3.419	114	3.478	147	3.520	180	3.554
16	3.048	49	3.316	82	3.421	115	3.480	148	3.521	181	3.555
17	3.064	50	3.320	83	3.423	116	3.481	149	3.522	182	3.556
18	3.077	51	3.325	84	3.425	117	3.482	150	3.523	183	3.556
19	3.090	52	3.329	85	3.427	118	3.484	151	3.524	184	3.557
20	3.103	53	3.333	86	3.429	119	3.485	152	3.525	185	3.558
21	3.115	54	3.337	87	3.431	120	3.486	153	3.526	186	3.559
22	3.127	55	3.341	88	3.434	121	3.488	154	3.527	187	3.560
23	3.138	56	3.344	89	3.436	122	3.489	155	3.528	188	3.561
24	3.148	57	3.348	90	3.439	123	3.490	156	3.529	189	3.561
25	3.158	58	3.351	91	3.440	124	3.492	157	3.531	190	3.562
26	3.167	59	3.355	92	3.442	125	3.493	158	3.532	191	3.563
27	3.176	60	3.358	93	3.444	126	3.494	159	3.533	192	3.564
28	3.186	61	3.362	94	3.445	127	3.496	160	3.534	193	3.565
29	3.195	62	3.365	95	3.447	128	3.497	161	3.535	194	3.565
30	3.204	63	3.368	96	3.449	129	3.498	162	3.536	195	3.566
31	3.212	64	3.372	97	3.451	130	3.500	163	3.537	196	3.567
32	3.220	65	3.375	98	3.453	131	3.501	164	3.538	197	3.568
33	3.229	66	3.378	99	3.455	132	3.502	165	3.539	198	3.569
34	3.237	67	3.381	100	3.457	133	3.503	166	3.540	199	3.570
35	3.245	68	3.384	101	3.459	134	3.505	167	3.541	200	3.570
36	3.251	69	3.388	102	3.460	135	3.506	168	3.542	240	3.605
37	3.257	70	3.391	103	3.462	136	3.507	169	3.543	300	3.632
38	3.262	71	3.393	104	3.463	137	3.508	170	3.544	360	3.654
39	3.268	72	3.396	105	3.465	138	3.510	171	3.545	420	3.672
40	3.274	73	3.398	106	3.467	139	3.511	172	3.546	480	3.687

Figure 13: 1% ANOX Scaling Factors

APPENDIX TWO: PROGRAMMING THE ANOX TABLES

For programming purposes, when the number of values, k, is between 18 and 360, two-decimal place approximations for the ANOX scaling factors in Figures 11, 12, and 13 may be obtained using equations of the following form with the coefficients in Figure 14.

$$ANOX_{\alpha} \approx a + b \{ 1 - \exp(-c\,k) - d\,[\,1 + \{ c \exp[-(d+e)\,k\,] - (d+e) \exp(-c\,k) \}/(d+e-c)\,]/(d+e) \}$$

	10% ANOX	5% ANOX	1% ANOX
a	1.694992405	1.908167737	2.563625622
b	9.847765969	9.766223275	6.253291710
c	0.006566458	0.006860473	0.007268872
d	0.059028250	0.063342224	0.053872041
e	0.011291737	0.011612903	0.011802217

Figure 14: Coefficients for Programming the ANOX Tables

The use of these equations to estimate the ANOX scaling factors will result in limits that will have approximately the stated risk of a false alarm. These equations are not suitable for $k < 18$, or for $k > 360$, because they give values that differ from the tabled values by two or more units in the second decimal place. Thus, while a table lookup will be needed for $k = 8$ to 17, the equations above can be used to approximate the ANOX limits for k up to 360.

In practice, it will be a rare event for a data set consisting of several hundred values to be found to be homogeneous. Remember the ANOX scaling factors define limits that will be exceeded by the *maximum* or *minimum* value either 1, 5, or 10 percent of the time. As more values fall outside the ANOX limits, and as these values fall further outside the limits, the evidence of a lack of homogeneity grows ever stronger.