The Three-Way Chart

Some data structures need more than an average and range chart

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The average and range chart handles most situations where the data can be logically organized into homogeneous subgroups. However, this chart breaks down when faced with a hierarchical data structure containing two or more levels of routine variation. For these situations I developed the three-way chart in 1982.

RATIONAL SUBGROUPING

To understand the three-way chart we need to begin with a review of the principles of rational subgrouping. All process behavior charts depend upon rational subgrouping and rational sampling. The reason for this goes back to one of the principles of understanding data: No data have meaning apart from their context. You cannot create meaning by using any mathematical computation. Computations can only distill out the meaning invested in your data by their context. Rational sampling and rational subgrouping are the keys to this distillation. If your computations do not respect the structure created within your data by their context, then your results will be meaningless.

The average and range chart is built on the foundation of rational subgrouping. To paraphrase Walter Shewhart, **when we place two values together in the same subgroup we are making a *judgment* that those two values were obtained under essentially the same conditions**. It is this element of judgment that makes our subgrouping rational. Without such judgment our subgrouping may well be irrational. As we take this idea and unwrap it we end up with several different ways to describe rational subgrouping.

Never knowingly subgroup unlike things together. Each subgroup must be logically homogeneous. When you combine apples and oranges and bananas together you may get a good fruit salad, but you will have lousy subgroups.

Minimize the variation within the subgroups. It is the variation within the subgroups that defines the background noise level. Any signals will have to be found against this background of noise. By minimizing the variation within the subgroups you will maximize the sensitivity of the process behavior chart.

Maximize the opportunity for variation between the subgroups. This requires thought about what types of potential signals might occur within your data stream. If you wish to compare two things they will need to be placed in different subgroups. If there is any reasonable possibility that two things might be different, they belong in different subgroups.

Do not bury signals within the subgroups. Subgrouping is effective only to the extent that the subgroups remain homogeneous. In many areas of statistics, where the estimation of parameters is the objective, larger amounts of data are preferred. This is not the case with average and range charts. You can destroy the homogeneity of the subgroups by increasing the size of the subgroups. Since the computations explicitly assume internal homogeneity for the subgroups, the logical homogeneity of the subgroups is vastly more important than the subgroup size.

Respect the context for your data. The context defines the structure within your data and is the key to discovering the assignable causes within your process changes. Even the order of the subgroups can be important. This is why we commonly use the time order for the chart. However, we can use other orders when they are meaningful in the context of the data.

THREE WAY CHARTS

The three-way chart is simply an extension of these principles of rational subgrouping. As suggested above, the key to having an effective chart is to compute limits that filter out the appropriate sources of routine background variation. When the data have more than one level of background variation we cannot always use the within-subgroup variation to compute appropriate limits for the subgroup averages. Our first example will be coated paper.

Paper is coated with a plastic film by means of an extrusion process. At the end of each roll three samples are taken and the coating weight is measured for each. The data for fifteen consecutive rolls are shown in Figure 1 organized into 15 subgroups of size three.

Roll No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Position															
Near Side	269	274	268	280	288	278	306	303	306	283	279	285	274	265	269
Middle	306	275	291	277	288	288	284	292	292	303	300	279	278	278	276
Far Side	279	302	308	306	298	313	308	307	307	297	299	293	297	282	286
Averages	284.7	283.7	289.0	287.7	291.3	293.0	299.3	300.7	301.7	294.3	292.7	285.7	283.0	275.0	277.0
Ranges	37	28	40	29	10	35	24	15	15	20	21	14	23	17	17

Figure 1: Film Coating Weights

What source of variation is found *within* the fifteen subgroups in Figure 1? The variation within each subgroup is the variation from position to position. This is variation is commonly referred to as variation "across the roll." Because of edge effects in the extrusion process it is expected that the coating weights will tend to be lighter on each side than in the middle. This across-the-roll variation is summarized by the ranges shown in Figure 1.

What source of variation is found *between* the subgroups in Figure 1? As we go from subgroup to subgroup we are looking at consecutive rolls, so the variation between subgroups is the roll-to-roll variation. This is commonly referred to as variation "down the web."

So here is the question of rational subgrouping: Is the variation across the roll the proper background variation for judging the variation from roll to roll? Logically the variation across the roll comes from a completely different source than the variation roll to roll. So how might you measure and track the roll-to-roll variation?

Another aspect of the data in Figure 1 pointed out to me by Matt Buzzeo is the difference in time intervals spanned. The three positions record simultaneous events while the roll-to-roll averages represent extended periods of time.

Differences such as these lead me to create the Three-Way Chart. When the variation within the subgroups is not the proper yardstick for constructing limits for the subgroup averages it is necessary to find another way of constructing reasonable limits. This is can be done by using a two-point moving range based upon the subgroup averages. The moving ranges will track the variation from one subgroup to the next (roll to roll), while the ranges within the subgroups will track the within-subgroup consistency (across the roll). Figure 2 shows the data of Figure 1 with the moving ranges added, and Figure 3 shows the resulting Three-Way Chart.

Roll No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Position															
Near Side	269	274	268	280	288	278	306	303	306	283	279	285	274	265	269
Middle	306	275	291	277	288	288	284	292	292	303	300	279	278	278	276
Far Side	279	302	308	306	298	313	308	307	307	297	299	293	297	282	286
Averages	284.7	283.7	289.0	287.7	291.3	293.0	299.3	300.7	301.7	294.3	292.7	285.7	283.0	275.0	277.0
Ranges	37	28	40	29	10	35	24	15	15	20	21	14	23	17	17
Moving Ranges		1.00	5.33	1.33	3.67	1.67	6.33	1.33	1.00	7.33	1.67	7.00	2.67	8.00	2.00

Figure 2: Film Coating Weights with Moving Ranges

Because the Three-Way Chart can hide signals if it is used improperly, it should be used only when there are multiple levels of background variation present in the data. Its use must be justified by the structure of the data and the structure of the subgroups.

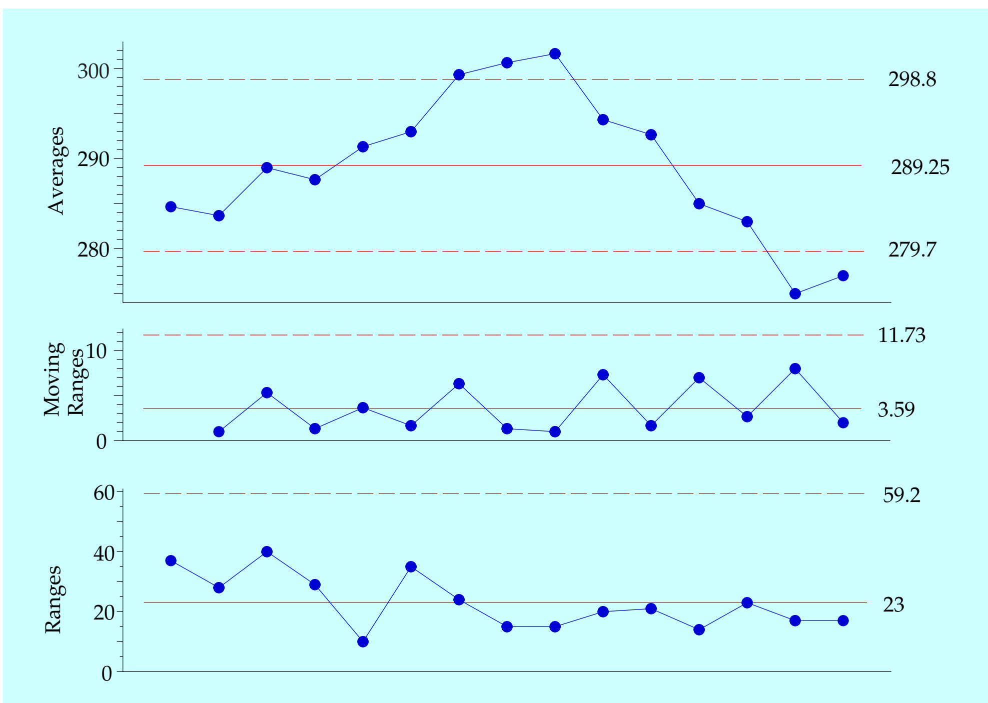

Figure 3: Three-Way Chart for Film Coating Weights

The Grand Average is 289.25. The Average Range is 23.0. The Average Moving Range is 3.59. The limits for the subgroup averages are:

$$289.25 \pm 2.66\,(3.59) = 279.7 \text{ to } 298.8$$

The upper limit for the moving ranges is:

$$3.268\,(3.59) = 11.73$$

The upper limit for the subgroup ranges is:

$$2.574\ (23.0)\ =\ 59.2$$

The Average Chart in Figure 3 shows that the coating weights differ from roll to roll. There is something about this process that is allowing the film thickness to vary excessively over time. The weak but persistent sawtooth pattern on the Moving Range Chart suggests that the larger changes in film thickness tend to occur every other roll. The chart for subgroup ranges shows that the variation in film thickness across the roll appears to be consistent. However, this position-to-position variation is greater than the variation down the roll. This is yet another area where there is potential for improvement.

SEPARATE CHARTS

When insight is needed to discover the assignable causes that affect your process it is usually profitable to disaggregate your data. Instead of subgrouping the Film Coating Weights by roll, we could use the data for each position to create three separate *XmR* charts. Here we ignore the question of across the roll variation, but gain insight into how the process is changing from roll to roll. (The limits were based on median moving ranges for greater sensitivity in this case.)

Roll No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Near Side	269	274	268	280	288	278	306	303	306	283	279	285	274	265	269
Moving Ranges		5	6	12	8	10	28	3	3	23	4	6	11	9	4
Middle	306	275	291	277	288	288	284	292	292	303	300	279	278	278	276
Moving Ranges		31	16	14	11	0	4	8	0	11	3	21	1	0	2
Far Side	279	302	308	306	298	313	308	307	307	297	299	293	297	282	286
Moving Ranges		23	6	2	8	15	5	1	0	10	2	6	4	15	4

Figure 4: Film Coating Weights

For the Near Side: Average = 281.8, Median Moving Range = 7.0, giving limits of:

$$281.80\ \pm\ 3.145\ (7.0)\ =\ 259.79 \text{ to } 303.82$$
$$\text{and } 3.865\ (7.0)\ =\ 27.1$$

For the Middle: Average = 287.133 Median Moving Range = 6.0, giving limits of:

$$287.13\ \pm\ 3.145\ (6.0)\ =\ 268.26 \text{ to } 306.00$$
$$\text{and } 3.865\ (6.0)\ =\ 23.2$$

For the Far Side: Average = 298.8 Median Moving Range = 5.5, giving limits of:

$$298.80\ \pm\ 3.145\ (5.5)\ =\ 281.50 \text{ to } 316.10$$
$$\text{and } 3.865\ (5.5)\ =\ 21.3$$

Inspection of Figure 5 shows that while all three positions are changing over time, most of the excursions shown in Figure 2 are due to the near side and far side coating weights. In addition, the occasional large moving ranges suggest that there are periodic upsets in the film thickness which occur at the different positions.

Whether the Three-Way chart or the three *XmR* charts will prove to be the most useful will depend upon the purpose of the charts, the way the charts are being used, and the control mechanisms available for this process. Context not only defines the appropriate organization for the data, but also guides the interpretation of the charts.

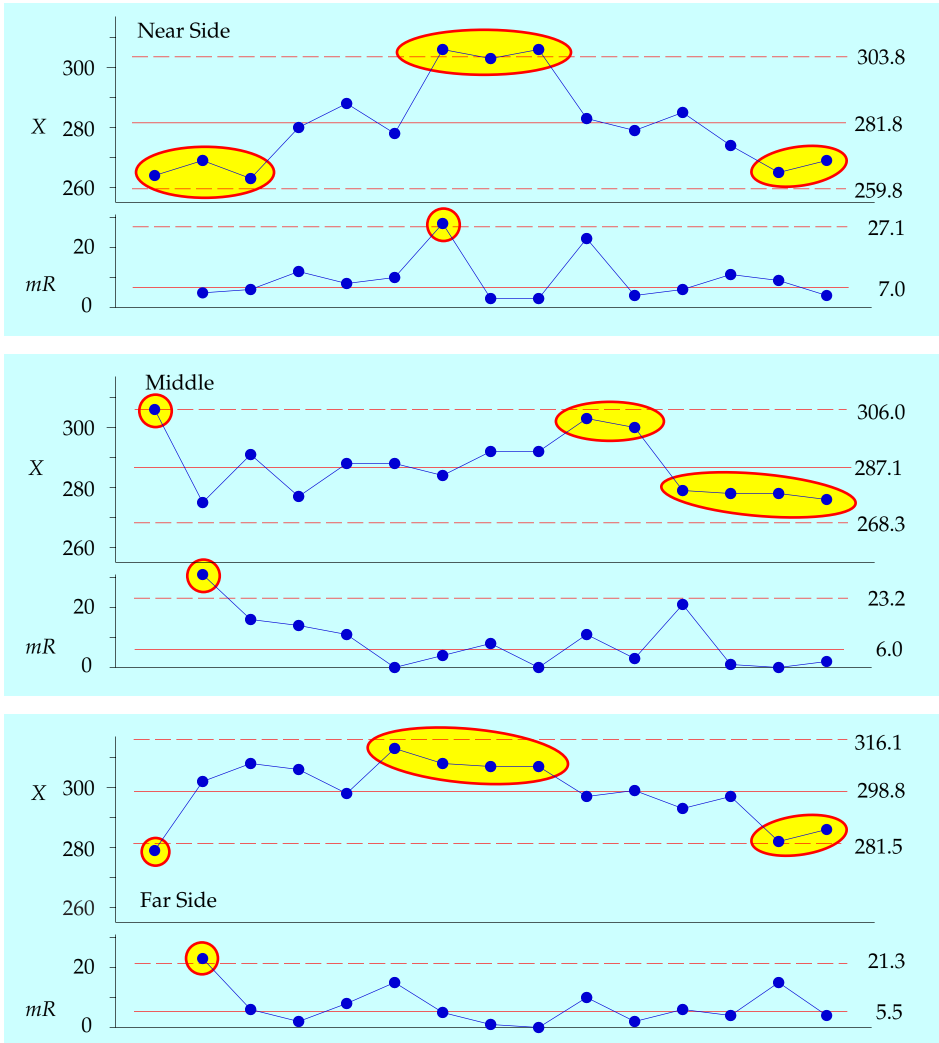

Figure 5: *XmR* Charts for Film Coating Weights by Position

In this example the use of the Three-Way Chart resulted in much tighter limits on the average chart than we would have found based on the subgrouping shown in Figure 1 (±9.5 versus ±40.8). This gain in sensitivity came from recognizing that we had two different types of routine background variation present in the data. Sometimes the Three-Way Chart works the other way round, as may be seen in the next example involving chemical batches in solution.

STEEL TENSILE STRENGTHS

Steel is being produced in 400 ton heats. Each heat is teemed into 40 ten-ton ingots. As this happens a sample is taken from ingots 1, 10, 20, 30, and 40 and these samples are tested for tensile strength. Thus we end up with five tensile strength values for each heat. The data in Figure 6 are for 25 consecutive heats of the same grade of steel. These tensile strengths are measured and recorded to the nearest 500 pounds, and are expressed in units of 1000 pounds.

	Tensile Strengths					$\bar{X}$	R		Tensile Strengths					$\bar{X}$	R
1	52.0	52.0	52.5	52.5	52.5	***52.30***	***0.5***	*14*	55.5	56.0	55.5	55.5	56.5	***55.80***	***1.0***
2	56.0	55.5	56.0	55.5	55.5	***55.70***	***0.5***	*15*	52.5	52.5	52.5	52.5	52.5	***52.50***	***0.0***
3	54.0	54.0	53.5	54.0	58.0	***54.70***	***4.5***	*16*	50.5	50.5	50.5	50.5	51.0	***50.60***	***0.5***
4	58.0	58.5	58.0	58.0	58.0	***58.10***	***0.5***	*17*	53.5	53.5	53.5	53.0	53.5	***53.40***	***0.5***
5	56.5	57.0	57.0	55.0	57.5	***56.60***	***2.5***	*18*	55.5	55.5	56.0	56.0	56.0	***55.80***	***0.5***
6	49.5	50.0	50.0	50.0	50.0	***49.90***	***0.5***	*19*	56.0	55.5	56.0	56.0	56.0	***55.90***	***0.5***
7	56.0	55.0	55.0	55.0	55.0	***55.20***	***1.0***	*20*	54.5	54.5	55.0	55.0	54.5	***54.70***	***0.5***
8	57.5	57.5	57.5	57.5	57.5	***57.50***	***0.0***	*21*	55.5	55.5	55.5	56.0	56.0	***55.70***	***0.5***
9	53.0	53.0	53.0	53.0	53.0	***53.00***	***0.0***	*22*	56.5	57.0	56.0	56.0	55.5	***56.20***	***1.5***
10	58.5	58.5	58.5	59.0	58.5	***58.60***	***0.5***	*23*	55.5	56.0	56.0	56.0	56.0	***55.90***	***0.5***
11	53.5	53.0	53.0	52.5	53.0	***53.00***	***1.0***	*24*	57.5	57.5	57.5	57.5	57.5	***57.50***	***0.0***
12	57.0	49.6	56.5	56.5	56.5	***55.22***	***7.4***	*25*	57.5	57.0	57.5	57.0	57.5	***57.30***	***0.5***
13	57.5	57.5	57.5	57.5	57.5	***57.50***	***0.0***								

Figure 6: Steel Tensile Strength Data

The Grand Average is 55.145, the Average Range is 1.016. With subgroups of size 5 the limits for the subgroup averages are 54.559 to 55.731, and the Upper Range Limit is 2.148. The Average and Range Chart is shown in Figure 7.

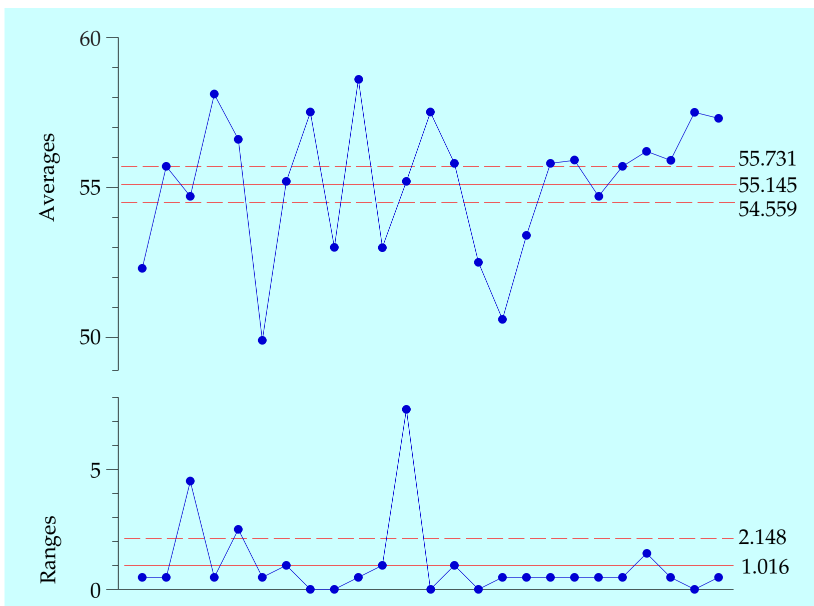

Figure 7: Average and Range Chart for Steel Tensile Data

The chart in Figure 7 has the variation within each heat isolated within each subgroup, while the heat-to-heat variation shows up between successive subgroups. Since each heat here is a highly agitated solution, it is unlikely that the variation within a heat will provide the right yardstick for judging the heat-to-heat variation. Not surprisingly, the chart shows that the heat-to-heat variation is much greater than the variation within the heats. The average chart limits based upon the within-heat variation do not allow us to properly assess the heat-to-heat variation. However, the range chart limits do properly assess the variation within the heats.

Using the subgroup averages to compute moving ranges we can obtain appropriate limits for the heat-to-heat variation. The average moving range between heats is 2.533.

$\bar{X}$	*mR*	$\bar{X}$	*mR*	$\bar{X}$	*mR*	$\bar{X}$	*mR*	$\bar{X}$	*mR*
52.30	—	49.90	6.7	53.00	5.6	50.60	1.9	55.70	1.0
55.70	3.4	55.20	5.3	55.22	2.22	53.40	2.8	56.20	0.5
54.70	1.0	57.50	2.3	57.50	2.28	55.80	2.4	55.90	0.3
58.10	3.4	53.00	4.5	55.80	1.7	55.90	0.1	57.50	1.6
56.60	1.5	58.60	5.6	52.50	3.3	54.70	1.2	57.30	0.2

Figure 8: Moving Ranges for the Average Tensile Strengths of Each Heat

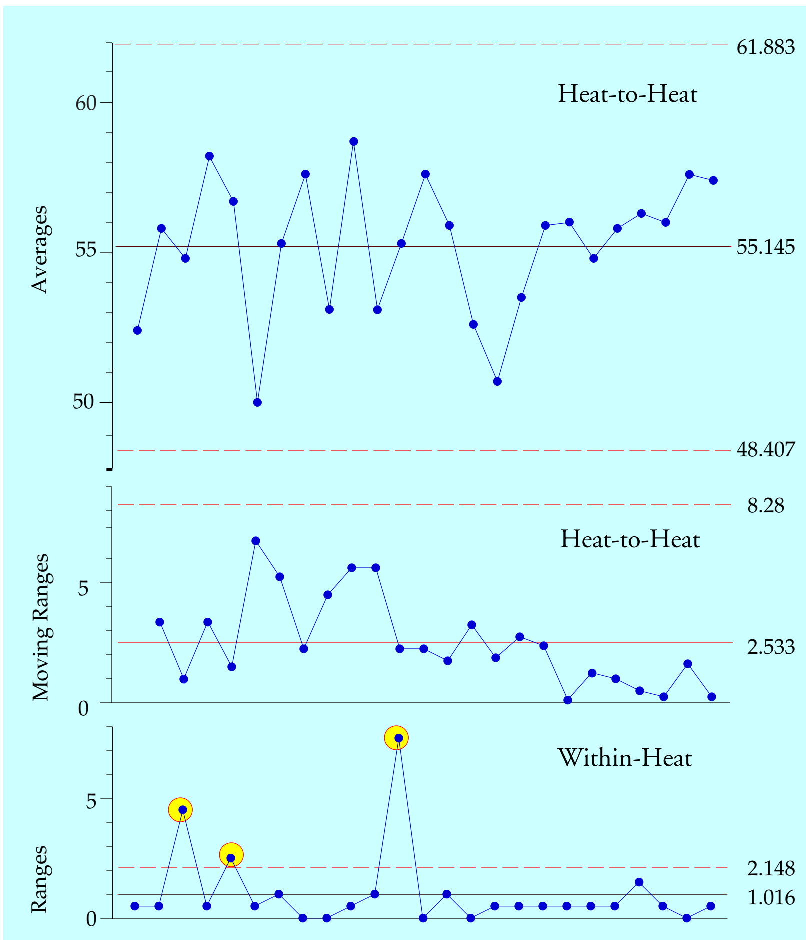

Figure 9: Three-Way Chart for the Steel Tensile Strengths

Heats 3, 5, and 12 show excessive within-heat variation, which could be important to know when rolling these ingots. Otherwise, these 25 heats show no evidence of exceptional variation from heat to heat or within each heat.

In effect the three-way chart takes the subgroup averages and treats these averages as raw data for an *XmR* chart. This utilizes the empirical nature of the *XmR* chart to obtain appropriate limits that allow for the routine variation which is not captured within the subgroups. The upper limit for the moving ranges is 8.28. If any moving range exceeded this limit, then that range would be identifying a sudden shift between heats, even if both averages remained within their limits.

In contrast with the example above, batches consisting of mixtures (rather than solutions) will often have more variation within a batch than batch-to-batch. Here the three way chart will tend to tighten the limits and make the average chart more sensitive. Batches of product with multiple values per batch, rolling processes, and web-based processes all tend to produce hierarchical data where there are two or more levels of routine variation present. When this happens ordinary average and range charts will not handle the hierarchy, but the Three-Way Chart will work.

This approach should not be used indiscriminately. The moving range calculated from the subgroup averages should only be used when the physical situation warrants its use. A Three-Way Chart should only be used when the data contain two or more inherently different sources of routine background variation and there is some overriding reason for tracking both sources of routine background variation on a single chart. This technique must always be justified by an appeal to the context. It can never be justified by the behavior of the data alone. Remember, some processes are actually so severely unpredictable that the appropriate charts will look like Figure 7.

SUMMARY

Process behavior charts are exceedingly versatile. The key to unlocking this versatility is rational sampling and rational subgrouping. Of course this requires judgment to obtain and organize data so that they will answer the interesting questions regarding your process. And judgment requires experience and practice. There is no shortcut. Learn the principles of rational subgrouping, and study the examples of those who have used SPC successfully.

In the meantime, if you are uncertain about a particular subgrouping, play with your data using alternate organizations, or avoid subgroups entirely by using an *XmR* chart. As one supervisor told me, "Nothing can hide on that *XmR* chart. Every point has to sink or swim by itself." As long as successive values are logically comparable, it is hard to screw up the subgrouping on an *XmR* chart. The moving ranges will empirically track the variation present in the individual values. Thus, when we have transformed the original data in some rational manner we can use the *XmR* chart with these transformed values. Here we used them with end-of-roll averages and batch averages. The completely empirical nature of the *XmR* chart makes it sort of a universal chart that will handle situations that other approaches will not handle.

For more on how to create useful charts see my ***QDD*** articles "Rational Sampling" and "Rational Sampling" in June and July of 2015.