The Levey-Jennings Chart

How to get the most out of your measurement system

Donald J. Wheeler

The Levey-Jennings chart was created in the 1950s to answer questions about the quality and consistency of measurement systems in the chemical and process industries. This column will illustrate the fatal flaw in this technique and show a better way to track the consistency of your measurement systems. In addition it will describe how to quantify the actual resolution of your measurements.

For our example we will use data from page 20 of Shewhart's *Economic Control of Quality of Manufactured Product*. These data were collected as part of a research project on measuring the resistivity (in megohms) of an electrical insulator. Since this test was destructive, these measurements were made on samples cut from the same sheet of material. In this way the measurements were as close to multiple measurements of the same thing as destructive tests can be. The data are shown in Figure 2. The average for these 64 values is 4430.4 and the global standard deviation statistic is 532.3.

The Levey-Jennings chart plots these 64 values as a running record and adds a central line and two limits. The central line is generally taken to be the average value, although when testing a known standard the central line may be set at the accepted value for the standard. The limits are then placed at a distance of three times the standard deviation statistic on either side of the central line. Any point that falls outside these limits is taken as evidence of an inconsistency in the measurement process.

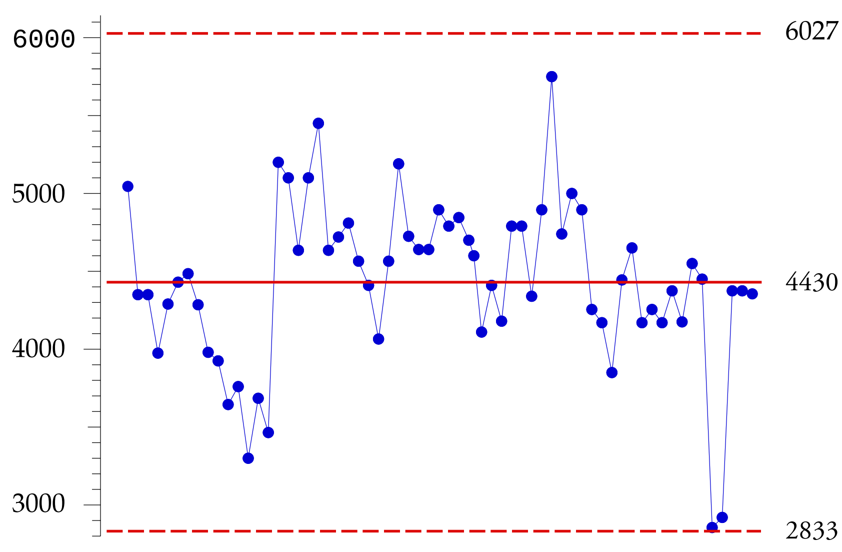

Figure 1: Original Levey-Jennings Chart for Resistivity Measurements of Figure 2

The Levey-Jennings chart for our data is shown in Figure 1. With no points outside the limits this measurement system is given a passing grade by the Levey-Jennings chart.

However, with the values drifting around and with the sudden changes in level, the running record of Figure 1 does not look like that of a consistent measurement process. So, we might well conclude that the original Levey-Jennings chart did not always detect problems with the measurement process. To remedy this weakness the Levey-Jennings chart was modified in 1981 by the addition of some additional criteria for detecting problems. These "Westgard rules" are slightly modified versions of the 1956 Western Electric zone tests that were published by James Westgard et. al. in the **Journal of Clinical Chemistry**.

X	*mR*	*X*	*mR*	*X*	*mR*	*X*	*mR*
5045		5100	100	4790	105	4170	85
4350	695	4635	465	4845	55	3850	320
4350	0	5100	465	4700	145	4445	595
3975	375	5450	350	4600	100	4650	205
4290	315	4635	815	4110	490	4170	480
4430	140	4720	85	4410	300	4255	85
4485	55	4810	90	4180	230	4170	85
4285	200	4565	245	4790	610	4375	205
3980	305	4410	155	4790	0	4175	200
3925	55	4065	345	4340	450	4550	375
3645	280	4565	500	4895	555	4450	100
3760	115	5190	625	5750	855	2855	1595
3300	460	4725	465	4740	1010	2920	65
3685	385	4640	85	5000	260	4375	1455
3463	222	4640	0	4895	105	4375	0
5200	1737	4895	255	4255	640	4355	20

Figure 2: 64 Original Measurements of Resistivity in Megohms

The Westgard rules are used to identify potential signals of a change in the measurement process whenever the one of the following conditions exists on the Levey-Jennings chart:

1. A point falls outside one of the three standard deviation limits;
2. Two successive points fall outside one of the two standard deviation lines;
3. Four successive points fall outside one of the one standard deviation lines;
4. Ten successive values all fall on the same side of the central line; or
5. Two successive values are on opposite sides of central line and are both beyond the two standard deviation lines.

The modified Levey-Jennings chart is shown in Figure 3 where rule four and then rule two indicate problems with these data.

Thus, the Westgard rules do improve the ability of the Levey-Jennings chart to detect changes in the measurement system. However, the whole process is still built on the global standard deviation statistic. As I explained in my columns for October 2013 and December 2015, it is always inappropriate to use a global standard deviation statistic when seeking to separate potential signals from probable noise. Whenever you use a global standard deviation statistic you are making a strong assumption that your data are homogeneous.

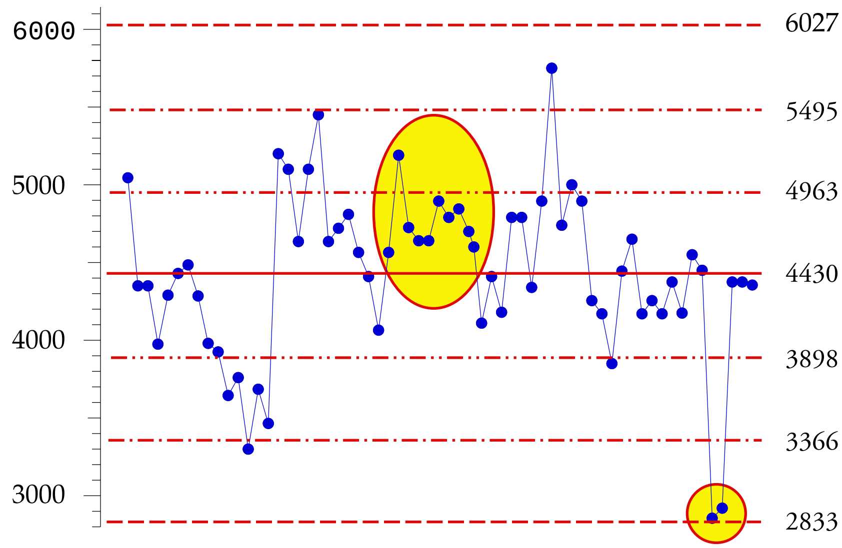

Figure 3: Modified Levey-Jennings Chart for Resistivity Measurements with Westgard Rules

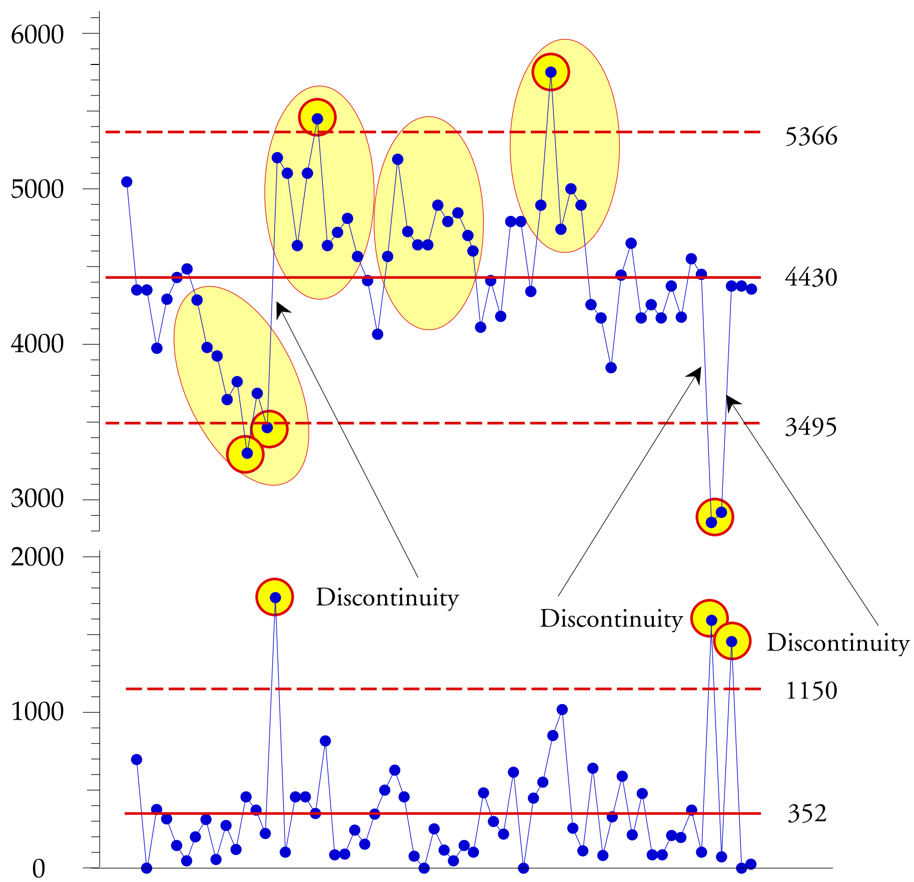

Figure 4: *XmR* Chart for Resistivity Measurements of Figure 2

At the beginning of the Twentieth Century we learned to avoid the use of global measures of dispersion when looking for potential signals within our data. This is why modern statistical techniques such as the analysis of variance, the analysis of means, and the process behavior chart all filter out the noise by using the *within subgroup* variation.

Thus, even though the Levey-Jennings chart was created in the 1950s, it was built upon a Nineteenth Century approach to analysis that has been known to be unsatisfactory and inappropriate for over 90 years!

PROCESS BEHAVIOR CHARTS

In Figure 4 we see the *XmR* chart for the 64 resistivity measurements of Figure 2. The average remains 4430.4, and the average moving range is 351.8. Dividing this latter value by d_2 = 1.128 we find our within-subgroup measure of dispersion to be *Sigma(X)* = 311.9. Thus, our three-sigma limits for the *X* chart are 3495 to 5366 and the upper limit for the range chart is 1150.

Here, in addition to the long run above the central line, we find six points and three moving ranges outside their limits. Including the points in the runs with the out-of-limits points, we find 34 of the 64 values to be associated with changes in the measurement process. Thus, we not only know that this measurement system is not producing consistent results, but we have clear indications about when the changes occurred.

So, while the original Levey-Jennings chart would mislead the researchers into feeling good about the resistivity measurements, the *XmR* chart makes it clear that these measurements are subject to some dominant assignable cause that makes this measurement process into a rubber ruler. As Shewhart reported regarding these data, the assignable cause was found and eliminated so that it could no longer take the measurement system on walkabout.

After making this change in the measurement process they proceeded to collect an additional 64 measurements of the resistivity of this same insulating material. These data are given in Figure 5. The *XmR* chart for these new data is shown in Figure 6. The average value is 4418 and the average moving range is 189.8. Thus *Sigma(X)* is 168.3 and these data show no evidence of inconsistency on the part of the measurement system.

X	*mR*	*X*	*mR*	*X*	*mR*	*X*	*mR*
4400		4465	150	4240	395	4310	90
4565	165	4375	90	4350	110	4250	60
4495	70	4660	285	4220	130	4315	65
4325	170	4260	400	4150	70	4380	65
4385	60	4675	415	4520	370	4490	110
4545	160	4800	125	4545	25	4240	250
4235	310	4410	390	4650	105	4350	110
4370	135	4675	265	4230	420	4320	30
4340	30	4260	415	4320	90	4475	155
4500	160	4000	260	4570	250	4725	250
4165	335	4360	360	4380	190	4540	185
4000	165	4500	140	4450	70	4565	25
4125	125	4460	40	4340	110	4210	355
4445	320	4360	100	4350	10	4540	330
4775	330	4265	95	4600	250	4600	60
4615	160	4635	370	4220	380	4850	250

Figure 5: 64 Additional Measurements of Resistivity in Megohms

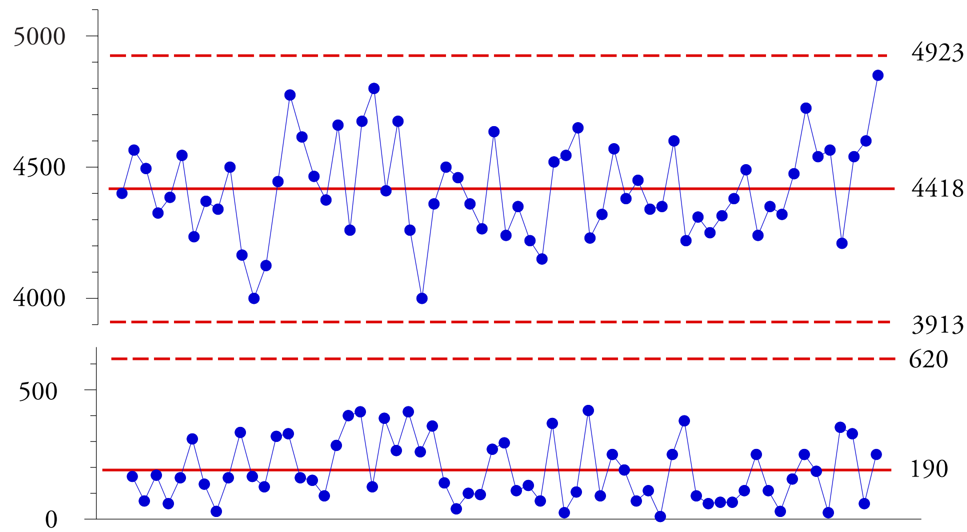

Figure 6: *XmR* Chart for Additional Resistivity Measurements of Figure 5

By identifying the assignable cause of inconsistency in this measurement system and then removing its effects they not only got rid of the outlying points of Figure 4, but they also reduced the overall measurement error by 48%. Now their measurement system is not only consistent but it is also operating with less measurement error than before.

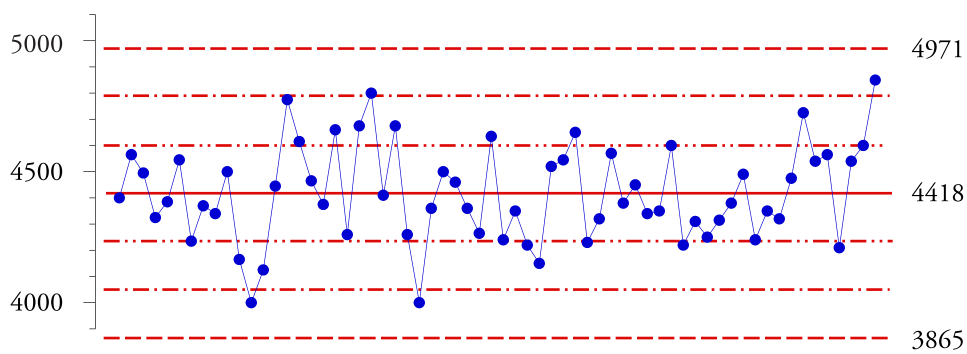

Figure 7: Levey-Jennings Chart for Additional Resistivity Measurements of Figure 5

The Levey-Jennings chart for the data of Figure 5 is shown in Figure 7. (The global standard deviation statistic for Figure 5 is 184.3 megohms.) With a consistent and predictable measurement system the Levey-Jennings chart will mimic the *XmR* chart. But when the measurement system is inconsistent the Levey-Jennings chart will be handicapped by the use of the global standard deviation statistic. This use of the global standard deviation statistic is the inherent fatal flaw in the Levey-Jennings chart. It is both primitive and naive. As a result the Levey-Jennings chart will only work with a good measurement process. Since it does not reliably detect problems with your measurement process the Levey-Jennings chart should not be used in practice.

THE DEMONSTRATED RESOLUTION OF THE MEASUREMENTS

How good are your measurements? How many digits should you record? To answer these questions we need to recap some of the history surrounding the problem of measurement error.

In 1805 Gauss proposed using a normal distribution as a model for the errors of measurement. In 1810 Laplace demonstrated why Gauss's assumption worked when he published a theorem known today as the Central Limit Theorem. After Lapace's theorem was published Bessel realized that the median error of a measurement could be characterized by 0.675σ, a quantity he called the probable error.

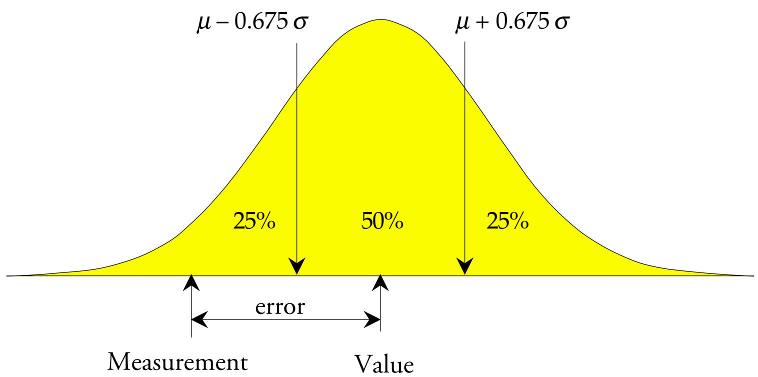

Figure 8: The Middle 50 Percent of a Normal Distribution Defines the Probable Error

Think of the mean of the distribution in Figure 8 as the value of an item being measured, and let the distribution represent a series of repeated measurements of that item. Here the standard deviation will characterize the measurement error. Half the time the measurements will be in the central region of Figure 8 and half the time they will fall in one of the two tails. While we typically do not know the value of the item to be measured, we can still think about the error of a single measurement as the difference between that measurement and the value of the item.

Putting all the elements of Figure 8 together, we can say that half the time a measurement will err by more than one probable error, and half the time a measurement will err by less than one probable error. Thus, the probable error defines the median error of a measurement. And if we are going to err by one probable error or more half the time, it does not pay to interpret a measurement more precisely than plus or minus one probable error. Thus, the probable error of a measurement system defines the effective and demonstrated resolution of the measurements.

In Figure 6 the average moving range for the measurement system was 189.8 megohms. Dividing by the bias correction factor d_2 = 1.128 we obtain a within-subgroup estimate of measurement error of *Sigma(X)* = 168.3 megohms. The probable error of this measurement process is then estimated by multiplying by the conversion factor of 0.675:

Probable Error of Resistivity Measurements = 0.675 x 168 megohms = 114 megohms.

While the data in Figure 5 are recorded to the nearest 5 megohms, they are only good to the nearest 114 megohms. Half the time these measurements will err by 114 megohms or less, and half the time these measurements will err by 114 megohms or more. These measurements have a demonstrated resolution of 114 megohms. This is the essential uncertainty attached to every measurement of resistivity in Figure 5.

When the measurement increment is appreciably larger than 114 megohms, the round-off will degrade the measurements. When, as is the case here, the measurement increment is substantially smaller than 114 megohms, then the users will be writing down pure noise in the last digit. A guideline for writing down the appropriate number of digits is to use a measurement increment that falls somewhere between the smallest effective measurement increment and the largest effective measurement increment, where:

Smallest Effective Measurement Increment = 0.2 x Probable Error

Largest Effective Measurement Increment = 2 x Probable Error

In this case they should use a measurement increment somewhere between 23 megohms and 227 megohms. So while they recorded these resistivities to the nearest 5 megohms, they could have rounded them off to the nearest 100 megohms without any serious degradation. To illustrate this the data of Figure 5 have been rounded to the nearest 100 megohms in Figure 10. The resulting *XmR* chart is shown in Figure 9.

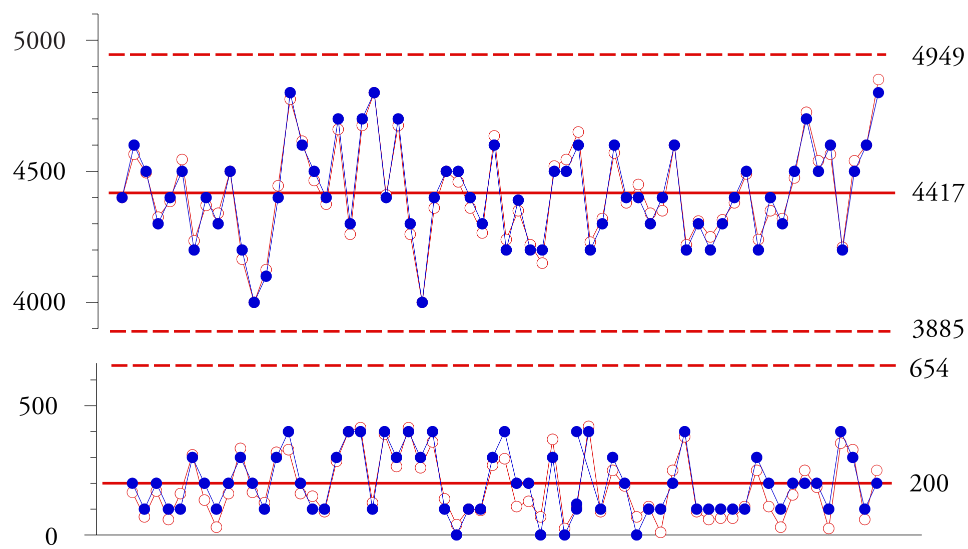

Figure 9: *XmR* Chart for Additional Resistivity Measurements Rounded to Nearest 100 Megohms

Figure 9 shows the *XmR* chart for the rounded resistivities of Figure 10. In addition it also shows the running records for the data of Figure 5 (in red). While the average moving range is slightly larger in Figure 9 than in Figure 6 both *XmR* charts tell the same story. These measurements are good to the nearest 114 megohms, and rounding to the nearest 100 megohms does not degrade the quality of the information they contain.

SUMMARY

So, while the Levey-Jennings chart will work with good data, it fails to work when the measurement system is operating inconsistently. It will not reliably tell you how to improve a measurement system that is not being operated up to its full potential. While the Westgard rules do help the Levey-Jennings chart to some degree, they cannot overcome the inherent fatal flaw of using a global measure of dispersion. For this reason, you should always avoid using a Levey-Jennings chart.

If you want to get the most out of your measurement processes you will need to use an *XmR* chart for repeated measurements of the same thing. Such a chart is known as a consistency chart. It will allow you to determine when extraneous factors influence your measurement process, so that you can identify them and control for their effects.

In addition, by using the within subgroup variation, the *XmR* consistency chart will provide you with a better estimate of the inherent measurement error than you can obtain from a Levey-Jennings chart where the global standard deviation will be inflated by any inconsistencies in the measurement process. Thus, the consistency chart allows you to quantify the demonstrated resolution of your measurements so that you will know how many digits to record.

X	*mR*	*X*	*mR*	*X*	*mR*	*X*	*mR*
4400		4500	100	4200	500	4300	100
4600	200	4400	100	4400	200	4200	100
4500	100	4700	300	4200	200	4300	100
4300	200	4300	400	4200	0	4400	100
4400	100	4700	400	4500	300	4500	100
4500	100	4800	100	4500	0	4200	300
4200	300	4400	400	4600	100	4400	200
4400	200	4700	300	4200	400	4300	100
4300	100	4300	400	4300	100	4500	200
4500	200	4000	300	4600	300	4700	200
4200	300	4400	400	4400	200	4500	200
4000	200	4500	100	4400	0	4600	100
4100	100	4500	0	4300	100	4200	400
4400	300	4400	100	4400	100	4500	300
4800	400	4300	100	4600	200	4600	100
4600	200	4600	400	4200	400	4800	200

Figure 10: 64 Additional Measurements of Resistivity in Megohms Rounded Off