The Analysis of Experimental Data

Minimizing the complexity improves communication

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You have spent good money obtaining your experimental results, and now the time has come to communicate those results to those who need to take action. This column will describe how to cut through the complexities of your analysis and communicate the results quickly and easily.

You have 30 seconds to make your point before your boss's eyes glaze over, and the easiest way to beat this 30 second rule is to use a graph. In thinking about how to beat the 30 second rule Ellis Ott decided to combine the graphic power of the average and range chart with the sensitivity of the analysis of variance. The result was the analysis of means (ANOM). The original version in 1967 used approximate critical values to compute the ANOM decision limits. Over the years several investigators have worked on sharpening up and extending these decision limits. The tables given here are excerpted from the results of the latest such revision carried out over the past two years. However, before we get to the ANOM, we shall begin with the traditional ANOVA.

THE ANALYSIS OF VARIANCE (ANOVA)

We begin with the data for a two-factor experiment. Two concentrations and five processing temperatures are examined for their effects upon the reflectivity of a certain coating. Three samples are coated and processed using each of the ten combinations of concentration and temperature. The data are shown in Figure 1. When these data are placed in an appropriate software package you will get the ANOVA table of Figure 2 as your output.

Concentration	5	5	5	5	5	10	10	10	10	10
Temperature	75	100	125	150	175	75	100	125	150	175
Measured	35	31	30	28	19	38	36	39	35	30
Reflectivity	39	37	31	20	18	46	44	32	47	38
	36	36	33	23	22	41	39	38	40	31
Averages	36.67	34.67	31.33	23.67	19.67	41.67	39.67	36.33	40.67	33.0
Ranges	4	6	3	8	4	8	8	7	12	8

Figure 1: The Reflectivity Data

Source	Sum of Squares	D.F.	Mean Squares	F-ratio	P-value
Concentration	616.5333	1	616.533	44.04	0.000 002
Temperature	591.2000	4	147.80	10.56	0.000 09
Interaction	196.1333	4	49.03	3.50	0.025
Within	280.0000	20	14.00		
Total	1683.8666	29			

Figure 2: The ANOVA Table for the Reflectivity Data

When using this ANOVA table to present your results you might tell your boss that both the concentration and the temperature have an effect upon the reflectivity, and that there is also an interaction effect as well. While this would be true, it would be incomplete. Three important questions that the ANOVA table does not answer are: How does the concentration level affect the reflectivity? How does the temperature affect the reflectivity? and How does the interaction effect change the reflectivity? While the ANOVA table tells us which effects are likely to be real, it does not show the actual effects. Other tools or techniques are needed to answer these questions. However, a simple ANOM chart will detect the effects and show the relationships at the same time.

ANOM and ANOR

The Analysis of Means is much simpler than the ANOVA. We compute the detection limits just like we compute limits for an average and range chart. The Grand Average for the data in Figure 1 is 33.733 and the average range is 6.80. With an overall alpha level of 10% the ANOM scale factor for k = 10 subgroups of size n = 3 from Table A is 0.893, so the ANOM detection limits are:

$$\textit{Grand Average} \pm \textit{ANOM}_{.10}\ (\textit{Average Range}) \;=\; 33.733 \pm 0.893\,(6.8) \;=\; 27.66 \text{ to } 39.81$$

The Analysis of Ranges (ANOR) proceeds in a similar manner. With an alpha level of 5%, and with k = 10 and n = 3, the ANOR scaling factor is found in Table B to be 2.519. Thus, the ANOR upper detection limit is:

$$\textit{ANOR}_{.05}\,(\,\textit{Average Range}) \;=\; 2.519\,(6.8) \;=\; 17.1.$$

The resulting ANOM plot is shown in Figure 3.

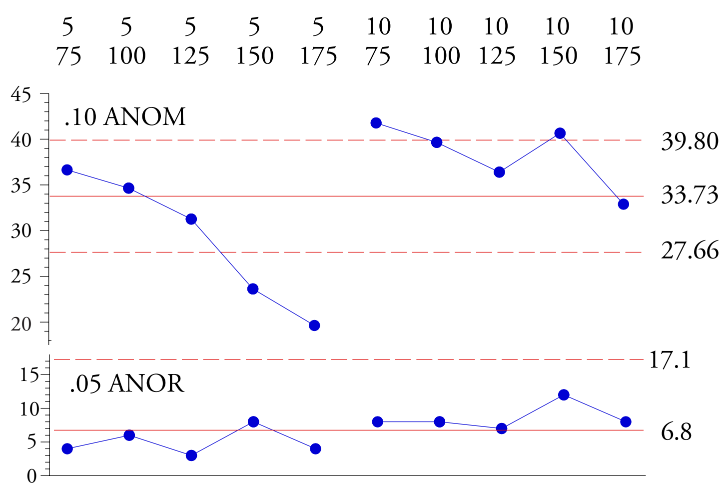

Figure 3: ANOM Chart for the Reflectivity Data

While Figure 3 looks like an average and range chart, and while it is as easy to construct as an average and range chart, it provides an analysis for your experimental data that uses your preferred alpha-level. In this case the ANOM uses an exploratory alpha level of 10%, while the ANOR uses a traditional alpha level of 5%. (With the usual three-sigma limits the ordinary average chart for these data would have an alpha-level of 4.2% while the range chart would have an alpha-level of 4%.) Since the graphs look the same, I am always careful to label my ANOM and ANOR charts as such and give the alpha level used.

In addition to being easy to construct, the ANOM chart is also easy to interpret. Four treatment averages are identified as being detectably different from the grand average, and there is no evidence of any differences in the within-treatment variation on the ANOR chart. By leaving a gap between points five and six we make it easy to see that the reflectivity goes up with concentration; the reflectivity goes down with temperature; and at the high concentration and higher temperatures there is an unexpected increase in reflectivity. This interaction effect can be exploited or avoided as necessary only after it has been visualized.

But how can we be sure that there are main effects for both concentration and temperature? These can be validated by using a follow-up analysis of main effects (ANOME).

ANOME

When a multifactor study is carried out using a fully-crossed design it will be easy to create main-effect charts for each of the factors in the study. In the example considered here the concentration factor has two levels: 5 grams per liter and 10 grams per liter. The ANOME for concentration will compare the average response for each of these two levels with the grand average. So the first step is to compute the average response for each level of concentration.

The average for the five treatments that had concentrations of 5 grams per liter is 29.20.

The average for the five treatments that had concentrations of 10 grams per liter is 38.27.

The ANOME scaling factor for comparing m = 2 averages with an alpha-level of 10% when the original study had k = 10 and n = 3 is found in Table C to be 0.184. The ANOME decision limits are:

$$\textit{Grand Average} \pm \textit{ANOME}_{.10}\,(\textit{Average Range}) = 33.73 \pm 0.184\,(6.8) = 32.48 \text{ to } 34.98$$

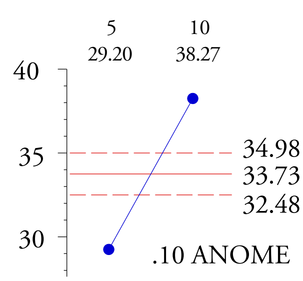

Figure 4: ANOME Chart for Concentration Main Effect

Clearly, there is a real effect here and the reflectivity goes up nine units on the average as the concentration goes from 5 to 10.

The temperature factor has five levels. The ANOME for temperature will compare the average response for each of these five levels with the grand average. So we compute the average response for each level of temperature.

The average for the two treatments that had temperatures of 75 degrees is 39.17.

The average for the two treatments that had temperatures of 100 degrees is 37.17.

The average for the two treatments that had temperatures of 125 degrees is 33.83.

The average for the two treatments that had temperatures of 150 degrees is 32.17.

The average for the two treatments that had temperatures of 175 degrees is 26.33.

The ANOME scaling factor for comparing $m = 5$ averages with an alpha-level of 10% when the original study had $k = 10$ and $n = 3$ is found in Table C to be 0.526. The ANOME decision limits are:

$$\textit{Grand Average} \ \pm \ \textit{ANOME}_{.10} \ (\textit{Average Range}) \ = \ 33.73 \ \pm 0.526\ (6.8) \ = \ 30.16 \text{ to } 37.31$$

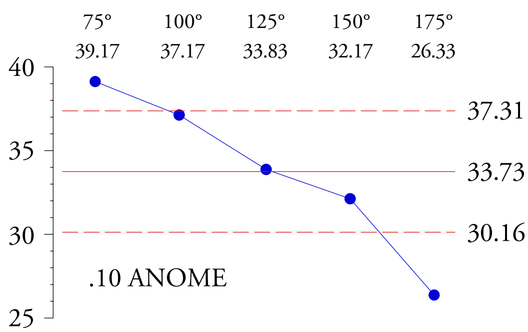

Figure 5: ANOME Chart for Temperature Main Effect

Clearly, there is a real effect here and the reflectivity goes down about 13 units on the average as temperature goes from 75 to 175.

Figures 4 and 5 confirm the impression given by Figure 3: Both concentration and temperature have an effect upon reflectivity. But how can we be sure about the interaction effect? The ANOVA is the easiest place to determine if a particular interaction effect is likely to be real, but the ANOM plot is the easiest place to interpret what the interaction effect means in terms of the experimental results. All interaction effects must show up on the ANOM plot as non-parallelness between corresponding line segments. Looking at the two curves from the ANOM plot in Figure 6, the two segments going from 75 degrees to 100 degrees are perfectly parallel, as are the two segments going from 100 to 125. However, the two segments going from 125 to 150 are almost perpendicular to each other. Finally the two segments going from 150 to 175 are reasonably parallel.

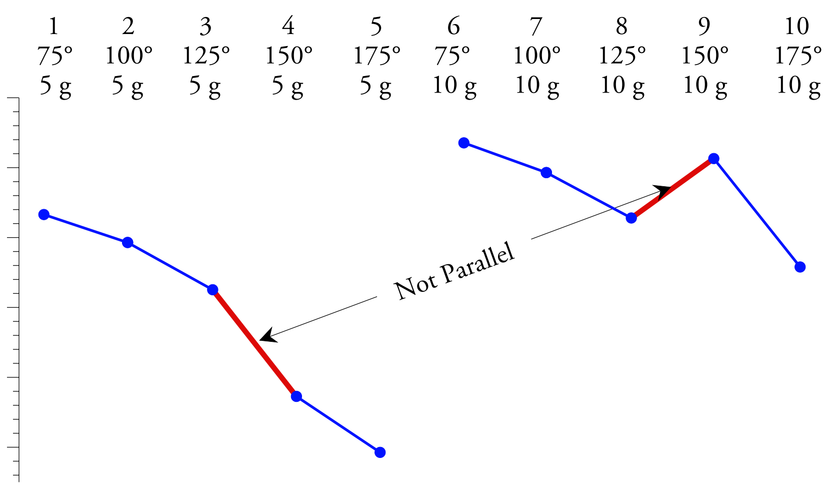

Figure 6: Interactions Must Show Up as Non Parallel Line Segments

Thus, the interaction effect detected in the ANOVA table consists of elevated reflectivities for the higher temperatures and high concentration treatment combinations (Subgroups 9 and 10).

SUMMARY

ANOM, ANOR, and ANOME allow you to analyze your experimental data with a minimum of effort. They provide you with a picture to use in presenting your results. And they allow you to focus the discussion on the interesting aspects of your data rather than getting bogged down in, or even misled by a discussion of the complexities of the analysis technique. This approach will minimize the time and effort required in order to begin to use your experimental data effectively. If you haven't started using ANOM, you simply do not know what you have been missing.

(This example comes from *Analyzing Experimental Data* © 2013 SPC Press. The following tables are excerpted from larger tables given there. Both the example and the tables are used with permission.)

Table A
Analysis of Means Scaling Factors

ANOM$_{.10}$

	n = 2	*n* = 3	*n* = 4	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9	*n*=10
k = 2	1.161	0.487	0.322	0.247	0.203	0.174	0.153	0.137	0.125
k = 3	1.515	0.678	0.455	0.351	0.289	0.248	0.219	0.197	0.179
k = 4	1.606	0.747	0.507	0.392	0.324	0.279	0.246	0.221	0.202
k = 5	1.656	0.791	0.540	0.419	0.347	0.299	0.264	0.237	0.217
k = 6	1.692	0.822	0.564	0.439	0.364	0.314	0.277	0.249	0.228
k = 7	1.719	0.844	0.582	0.453	0.376	0.325	0.287	0.258	0.236
k = 8	1.740	0.864	0.596	0.466	0.386	0.334	0.295	0.266	0.243
k = 9	1.758	0.879	0.609	0.476	0.395	0.342	0.302	0.273	0.248
k = 10	1.773	0.893	0.620	0.485	0.402	0.348	0.307	0.278	0.254
k = 11	1.787	0.905	0.629	0.492	0.409	0.354	0.312	0.283	0.257
k = 12	1.797	0.915	0.637	0.498	0.414	0.359	0.317	0.286	0.261

ANOM$_{.05}$

	n = 2	*n* = 3	*n* = 4	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9	*n*=10
k = 2	1.715	0.636	0.406	0.307	0.250	0.213	0.187	0.167	0.152
k = 3	2.009	0.829	0.543	0.414	0.339	0.290	0.255	0.228	0.208
k = 4	2.025	0.887	0.591	0.453	0.372	0.320	0.281	0.253	0.230
k = 5	2.027	0.922	0.620	0.477	0.394	0.339	0.298	0.268	0.244
k = 6	2.029	0.945	0.640	0.495	0.409	0.352	0.310	0.279	0.255
k = 7	2.031	0.963	0.656	0.508	0.420	0.363	0.319	0.288	0.263
k = 8	2.033	0.977	0.668	0.519	0.430	0.371	0.327	0.295	0.269
k = 9	2.036	0.990	0.679	0.528	0.438	0.378	0.333	0.301	0.274
k = 10	2.038	1.000	0.689	0.537	0.445	0.384	0.338	0.306	0.279
k = 11	2.041	1.009	0.697	0.543	0.450	0.390	0.343	0.310	0.283
k = 12	2.045	1.017	0.703	0.550	0.455	0.395	0.348	0.314	0.286

ANOM$_{.01}$

	n = 2	*n* = 3	*n* = 4	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9	*n*=10
k = 2	3.972	1.059	0.618	0.448	0.357	0.300	0.261	0.232	0.210
k = 3	3.630	1.216	0.749	0.556	0.448	0.380	0.331	0.295	0.268
k = 4	3.243	1.223	0.781	0.587	0.476	0.406	0.356	0.318	0.289
k = 5	3.037	1.227	0.797	0.604	0.494	0.422	0.369	0.332	0.302
k = 6	2.910	1.229	0.809	0.616	0.506	0.433	0.380	0.341	0.311
k = 7	2.825	1.231	0.817	0.625	0.515	0.441	0.388	0.349	0.318
k = 8	2.751	1.233	0.824	0.635	0.522	0.448	0.394	0.355	0.323
k = 9	2.700	1.235	0.830	0.640	0.526	0.453	0.400	0.359	0.328
k = 10	2.668	1.237	0.834	0.645	0.534	0.459	0.403	0.363	0.333
k = 11	2.641	1.239	0.841	0.651	0.537	0.463	0.408	0.367	0.335
k = 12	2.618	1.240	0.845	0.654	0.543	0.468	0.412	0.369	0.338

Table B
Analysis of Ranges Scaling Factors

$ANOR_{.10}$

	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n=10
k = 2	1.854	1.628	1.509	1.440	1.394	1.361	1.336	1.317	1.301
k = 3	2.252	1.850	1.680	1.586	1.523	1.480	1.447	1.422	1.401
k = 4	2.485	1.987	1.786	1.675	1.604	1.554	1.516	1.487	1.463
k = 5	2.649	2.085	1.862	1.740	1.661	1.606	1.565	1.532	1.507
k = 6	2.775	2.159	1.919	1.790	1.706	1.646	1.603	1.568	1.541
k = 7	2.877	2.218	1.966	1.829	1.741	1.678	1.633	1.596	1.567
k = 8	2.961	2.268	2.005	1.862	1.771	1.706	1.658	1.620	1.591
k = 9	3.030	2.310	2.038	1.890	1.796	1.728	1.680	1.641	1.610
k = 10	3.090	2.346	2.065	1.915	1.818	1.748	1.699	1.659	1.628
k = 11	3.146	2.377	2.092	1.935	1.837	1.766	1.715	1.675	1.642
k = 12	3.192	2.408	2.113	1.954	1.854	1.782	1.730	1.688	1.654

$ANOR_{.05}$

	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n=10
k = 2	1.924	1.725	1.598	1.519	1.466	1.427	1.399	1.376	1.357
k = 3	2.436	1.993	1.795	1.685	1.611	1.561	1.523	1.493	1.469
k = 4	2.720	2.145	1.911	1.782	1.699	1.640	1.595	1.562	1.534
k = 5	2.906	2.250	1.991	1.851	1.760	1.696	1.648	1.609	1.581
k = 6	3.044	2.329	2.051	1.903	1.806	1.737	1.687	1.648	1.616
k = 7	3.155	2.390	2.098	1.943	1.840	1.769	1.717	1.676	1.644
k = 8	3.241	2.440	2.138	1.974	1.871	1.798	1.743	1.701	1.668
k = 9	3.313	2.484	2.172	2.004	1.898	1.820	1.766	1.725	1.687
k = 10	3.374	2.519	2.199	2.029	1.919	1.841	1.784	1.740	1.705
k = 11	3.432	2.553	2.226	2.050	1.937	1.859	1.801	1.757	1.720
k = 12	3.479	2.581	2.248	2.068	1.955	1.875	1.817	1.770	1.732

$ANOR_{.01}$

	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n=10
k = 2	1.984	1.868	1.750	1.663	1.600	1.552	1.516	1.487	1.464
k = 3	2.722	2.264	2.023	1.882	1.789	1.721	1.672	1.634	1.601
k = 4	3.158	2.470	2.166	2.000	1.893	1.815	1.760	1.716	1.681
k = 5	3.423	2.594	2.260	2.078	1.960	1.880	1.818	1.770	1.733
k = 6	3.607	2.687	2.330	2.135	2.006	1.921	1.858	1.811	1.771
k = 7	3.744	2.753	2.380	2.178	2.045	1.958	1.893	1.845	1.801
k = 8	3.846	2.808	2.423	2.214	2.078	1.988	1.922	1.872	1.827
k = 9	3.924	2.854	2.455	2.244	2.105	2.013	1.945	1.893	1.849
k = 10	3.990	2.890	2.484	2.269	2.128	2.033	1.965	1.911	1.867
k = 11	4.048	2.924	2.509	2.289	2.149	2.051	1.984	1.927	1.882
k = 12	4.093	2.950	2.532	2.308	2.167	2.068	1.998	1.940	1.896

Table C
Analysis of Main Effects Scaling Factors

ANOME$_{.10}$

k	*m*	*n* = 2	*n* = 3	*n* = 4	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9	*n*=10
4	**2**	0.636	0.310	0.213	0.166	0.138	0.119	0.105	0.095	0.086
6	**2**	0.483	0.244	0.170	0.133	0.111	0.096	0.085	0.076	0.070
6	**3**	0.885	0.440	0.305	0.237	0.198	0.170	0.150	0.135	0.124
8	**2**	0.404	0.209	0.145	0.114	0.095	0.083	0.073	0.065	0.060
8	**4**	0.976	0.492	0.342	0.267	0.223	0.192	0.170	0.153	0.140
9	**3**	0.681	0.349	0.244	0.191	0.160	0.138	0.122	0.110	0.100
10	**2**	0.354	0.184	0.129	0.102	0.085	0.073	0.065	0.058	0.053
10	**5**	1.032	0.526	0.367	0.287	0.239	0.206	0.182	0.165	0.151
12	**2**	0.317	0.167	0.118	0.093	0.077	0.067	0.059	0.053	0.048
12	**3**	0.571	0.298	0.208	0.163	0.136	0.118	0.106	0.094	0.085
12	**4**	0.757	0.392	0.274	0.216	0.180	0.156	0.138	0.124	0.113
12	**6**	1.069	0.550	0.384	0.301	0.251	0.217	0.192	0.173	0.158

ANOME$_{.05}$

k	*m*	*n* = 2	*n* = 3	*n* = 4	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9	*n*=10
4	**2**	0.832	0.385	0.261	0.203	0.167	0.143	0.126	0.114	0.104
6	**2**	0.609	0.299	0.207	0.161	0.134	0.115	0.102	0.091	0.084
6	**3**	1.083	0.519	0.355	0.275	0.230	0.196	0.174	0.156	0.143
8	**2**	0.502	0.254	0.175	0.137	0.114	0.099	0.088	0.078	0.072
8	**4**	1.159	0.568	0.391	0.305	0.254	0.218	0.193	0.174	0.158
9	**3**	0.813	0.408	0.283	0.220	0.183	0.158	0.140	0.127	0.115
10	**2**	0.436	0.223	0.156	0.122	0.102	0.088	0.078	0.070	0.065
10	**5**	1.204	0.599	0.415	0.324	0.269	0.232	0.205	0.185	0.169
12	**2**	0.388	0.202	0.141	0.111	0.093	0.080	0.071	0.063	0.058
12	**3**	0.675	0.346	0.241	0.189	0.157	0.136	0.121	0.108	0.098
12	**4**	0.884	0.450	0.313	0.244	0.203	0.176	0.156	0.140	0.128
12	**6**	1.230	0.621	0.431	0.338	0.280	0.242	0.215	0.193	0.176

ANOME$_{.01}$

k	*m*	*n* = 2	*n* = 3	*n* = 4	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9	*n*=10
4	**2**	1.391	0.562	0.367	0.279	0.230	0.194	0.171	0.154	0.140
6	**2**	0.930	0.420	0.284	0.218	0.180	0.155	0.137	0.122	0.112
6	**3**	1.597	0.697	0.463	0.355	0.295	0.250	0.221	0.198	0.181
8	**2**	0.736	0.350	0.239	0.185	0.155	0.132	0.117	0.104	0.096
8	**4**	1.599	0.734	0.499	0.383	0.316	0.272	0.240	0.216	0.196
9	**3**	1.122	0.533	0.366	0.281	0.233	0.199	0.176	0.161	0.145
10	**2**	0.623	0.305	0.210	0.165	0.137	0.117	0.103	0.093	0.085
10	**5**	1.601	0.759	0.520	0.401	0.331	0.285	0.252	0.227	0.207
12	**2**	0.549	0.274	0.190	0.149	0.124	0.106	0.093	0.084	0.077
12	**3**	0.909	0.446	0.308	0.240	0.199	0.170	0.152	0.136	0.123
12	**4**	1.168	0.570	0.392	0.305	0.252	0.218	0.193	0.174	0.158
12	**6**	1.604	0.777	0.534	0.412	0.342	0.294	0.260	0.233	0.214