What Is the Zone of Economic Production?

And how can you get there?

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In my last two columns I showed how to convert capability indexes into the *Effective Cost of Production and Use* and how to use these costs to quantify the payback for various improvement options. In this column I will show how the *Effective Cost of Production and Use* defines what is required in order to operate in the zone of economic production.

Figure One shows how the *Effective Cost of Production and Use* varies with the capability ratio, C_p, for the case where all nonconforming product is scrapped. The first curve is labeled with a zero and represents the case where the process is perfectly centered within the specifications. The region to the left of this curve is the impossible zone and the region to the right is for situations where the process is not centered within the specifications.

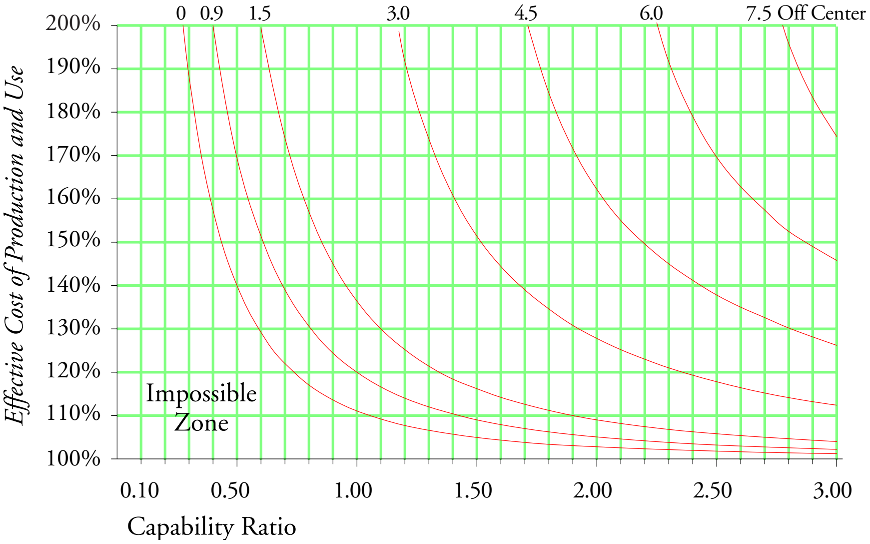

Figure One: How the *Effective Cost of Production and Use* Varies with the Capability Ratio

The second curve in Figure One is labeled 0.9. This curve represents the relationship between the *Effective Cost of Production and Use* and the capability ratio when the process is 0.9 sigma off center. The third curve, labeled 1.5, represents the relationship between the *Effective Cost of Production and Use* and the capability ratio when the process is 1.5 sigma off center. In a similar manner, the remaining curves show what happens when the process is 3.0 sigma off center, 4.5 sigma off center, etc. Each of these curves is a direct result of the rigorous mathematical argument presented in my column of August 2. As we move to the right along any one of these curves we see that they all flatten out as they approach the limiting value of 100% for the *Effective*

Cost of Production and Use.

This nonlinear behavior identifies a diminishing return. As the *Effective Cost of Production and Use* gets closer to 100%, the greater the increase in the capability ratio will need to be to see further reductions in the *ECP&U* value. Thus, it is reasonable to acknowledge this diminishing return by defining some region close to 100% as the zone where further improvements are no longer required. This will be the zone of economic production. I have chosen to draw this line at 110% in Figure Two. Some might prefer to draw this line higher or lower, but this value seems to me to be consistent with how we often act in practice.

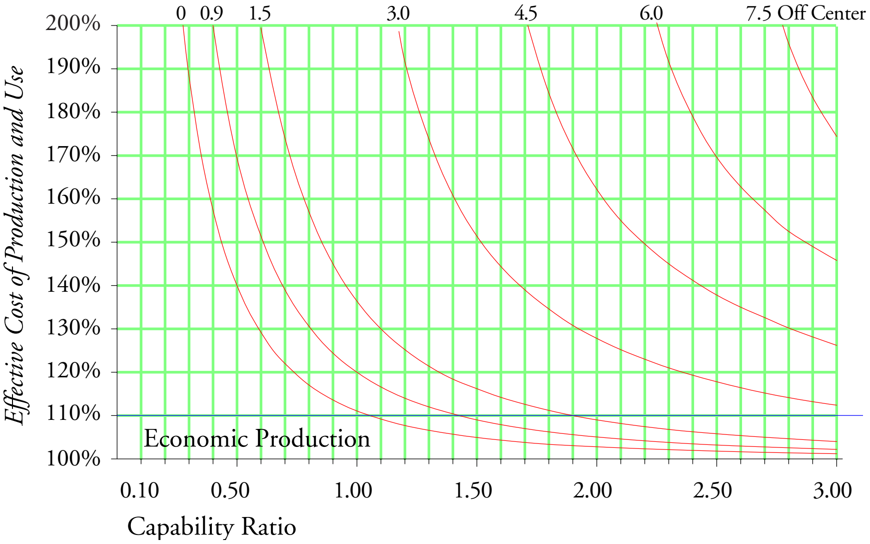

Figure Two: The Zone of Economic Production

From Figure Two we see that a process that is perfectly centered within the specifications will enter the zone of economic production when the capability ratio is about 1.05. A process that is operated 0.9 sigma off center will enter the zone of economic production when the capability ratio is about 1.4. A process that is operated 1.5 sigma off center will enter the zone of economic production when the capability ratio is about 1.9.

All of the results above can be computed with mathematical precision. Unfortunately, our processes are seldom so exact and precise. Processes change, and we have to face the consequences of these changes. So how do the mathematical results given above translate into practice? To answer this we need to consider how our ability to detect a change will depend upon the size of that change. Published tables of the power functions of various statistical procedures computed by this author and others have all shown that smaller shifts are always harder to detect than larger shifts. For a simple process behavior chart we can summarize this result as shown in Figure Three.

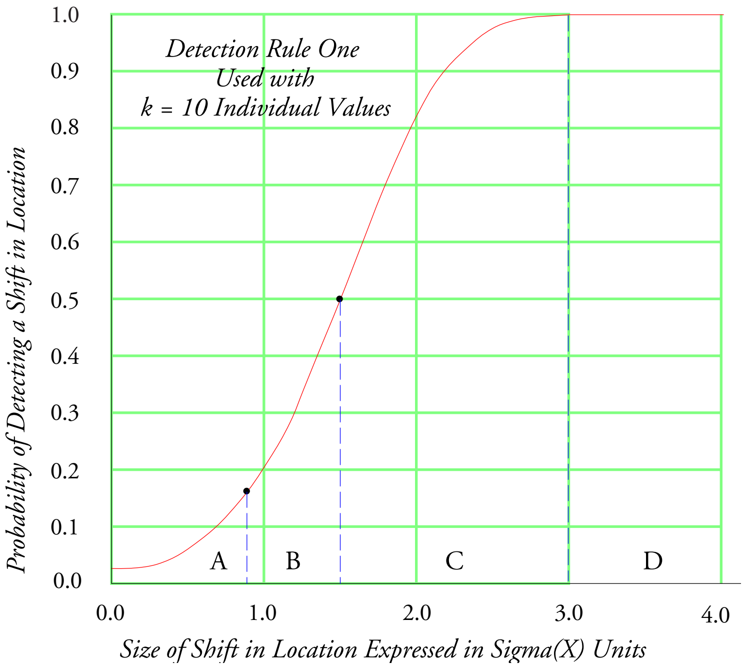

Figure Three: The Power Function for *X* Charts

The vertical scale in Figure Three shows the probability of detecting a shift in the process location within 10 observations following that shift. The horizontal scale shows the size of the process shift. Detection Rule One is a point falling outside the three-sigma limits of the *X* Chart.

When the shift size is zero, there is no shift and we want a small probability of a false alarm, as shown in Figure Three. In the region marked A the shift size is less than 0.9 sigma. In this region there will be a small probability of detecting a process shift. Shifts of this size will be hard to detect in a timely manner.

In the region B the size of the shift is between 0.9 sigma and 1.5 sigma. Here the probability of detecting a shift triples over what it was in region A. In this region we will be slow to detect our process changes.

In the region marked C the size of the shift is between 1.5 and 3.0 sigma. Here the probability of detecting our process shift rapidly increases up to a virtual certainty. Shifts of this size and larger will be rapidly and reliably detected. Figure Four shows how the regions of Figure Three relate to the curves of Figures One and Two.

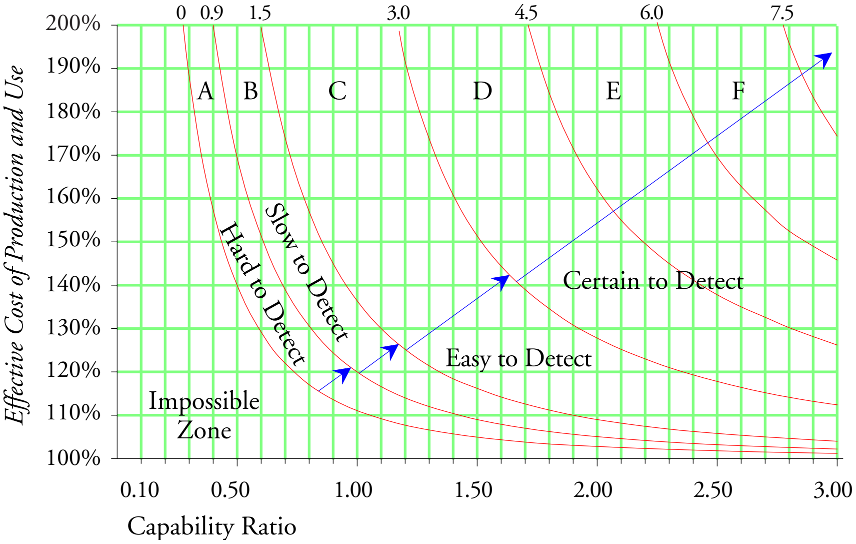

Figure Four: Our Ability to Detect Process Changes

No real process is ever exactly the same over time. Things change, and these changes will result in shifts in the product stream. Even when a process is operated predictably the many Common Causes of routine variation will buffet the process and shift the process stream around slightly. When we make allowance for this buffeting and our inability to detect small shifts in a timely manner, we have to conclude that when a process is operated predictably and on-target, it will generally inhabit region A, with occasional excursions into region B. Therefore, when a process is operated predictably and on-target, it is safe to say that the process average will be within 1.5 *Sigma(X)* of the target most of the time. When larger excursions occur they will generally be detected in a timely manner and the process can be returned to operating on-target.

But what happens when a process is not operated predictably? Part of the smoke and mirror computations of various six-sigma programs has been a completely unsupported assumption that an unpredictable process will not shift around more than plus or minus 1.5 *Sigma(X).* Real data completely refute this assumption.

Even a well-behaved process that is operated predictably and on-target will experience occasional excursions that are substantially greater than 1.5 *Sigma(X).* A good example of this is the Tokai Rika Cigar Lighter process used my column of August 2. (The complete chart is shown in Chapter Seven of Wheeler and Chambers, ***Understanding Statistical Process Control, Third Edition,*** SPC Press, 2010.) Even though this is one of the most predictable processes you will ever see, the complete chart shows several excursions in the 2 to 3 *Sigma(X)* range, and one excursion of 6.7 *Sigma(X).* If a reasonably predictable process can suffer repeated excursions of magnitude two-sigma, three-sigma, and even six-sigma, what hope would a truly unpredictable process have of always staying within 1.5 *Sigma(X)* of a target value?

In answer to this question consider the Batch Weight Data for one week's production which were given in my September 1 column. The *XmR* Chart for these data is shown in Figure Five. There we see a process that shifts around from 8 *Sigma(X)* below the average to 9 *Sigma(X)* above the average within a 24 hour time period. Moreover, on Friday afternoon, this process oscillates from more than 7 *Sigma(X)* above the average to more than 5 *Sigma(X)* below the average *for several successive batches*.

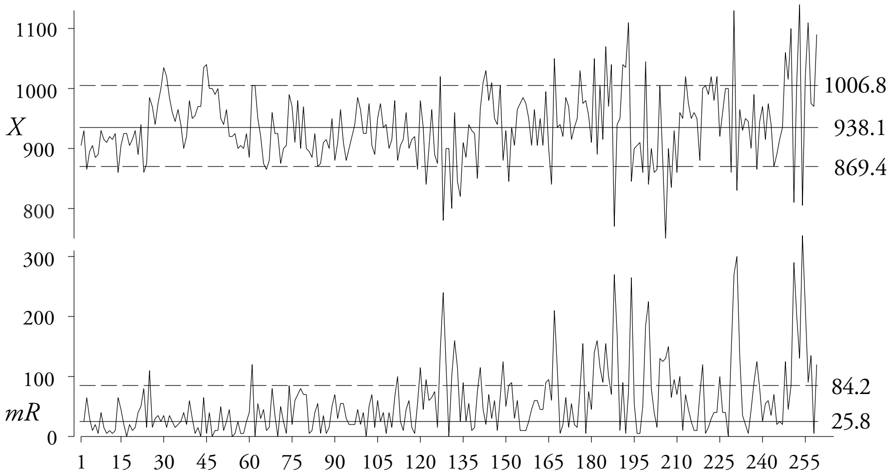

Figure Five: A Unpredictable Process Knows No Bounds

What part of the word unpredictable is not clear? *There is simply no way to place a bound on the size of excursions that may be seen when a process is operated unpredictably.* Hence, all computations, and all arguments, that are based on the assumption that an unpredictable process will not shift around more than ±1.5 *Sigma(X)* are completely spurious.

HOW TO OPERATE IN THE ZONE OF ECONOMIC PRODUCTION

When we combine the facts of life and the mathematical results given above, we end up with the picture shown in Figure Six. We simply cannot *begin* to operate within the zone of economic production until we have a process with a capability ratio that exceeds 1.10 that is operated predictably and on-target.

Making allowance for those hard-to-detect process excursions into region A, we cannot be *assured* of operation in the zone of economic production until we have a process with a capability ratio in excess of 1.50 that is operated predictably and on-target.

Making allowance for occasional undetected excursions into region B, we cannot *guarantee* operation in the zone of economic production until we have a process with a capability ratio in excess of 1.90 that is operated predictably and on-target.

Thus, with the rigorous approach to making sense of our capability indexes that I have outlined in this and my two previous columns, we find that the minimum requirements for assured operation within the zone of economic production are:

1. You will need to operate your process predictably.
2. You will need to operate your process on-target.
3. You will need to have a capability ratio in excess of 1.50.

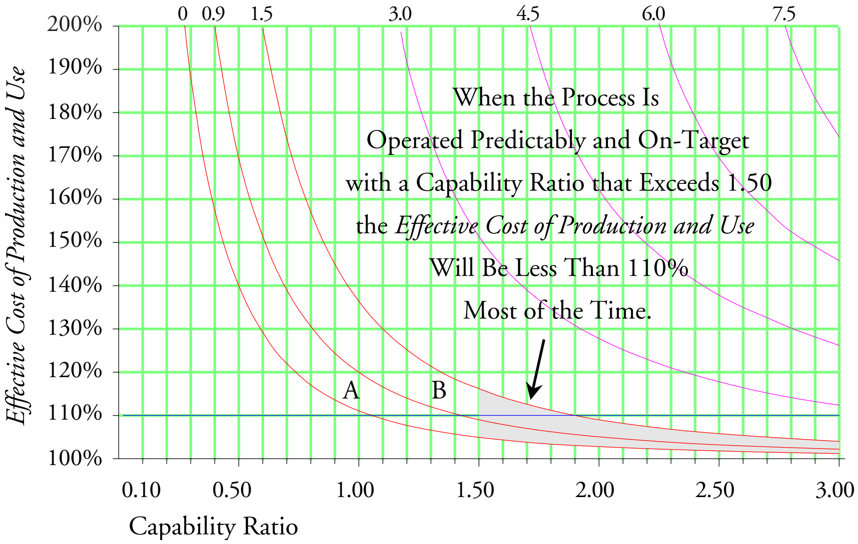

Figure Six: How to Operate in the Zone of Economic Production

Operating a process predictably is a necessity simply because this is the only way to get your process to operate up to its full potential. To operate a process predictably is to operate that process with minimum variance. Unpredictable operation will inevitably increase the variation, which will lower the capability indexes and increase the *Effective Cost of Production and Use*.

Operating on-target is a necessity simply because, regardless of how large your capability ratio might be, operating off-target can take you out of the zone of economic production.

Therefore, any attempt to define economic operation based on capability indexes alone is simply not sufficient. No matter how we might dress our capability indexes up as "process sigma levels" or talk about the "parts per million defective" or the "defects per million," we still cannot define economic operation without reference to predictable and on-target operation. Figure Six provides the first and only rigorous definition of what it takes to operate in the "six-sigma zone." It is what has been missing form many six-sigma programs.

SUMMARY

As I have shown in this and my two previous columns, the *Effective Cost of Production and Use* allows us to convert capability indexes and performance indexes into the language of management. Moreover, when we follow the mathematics behind this conversion to its logical conclusion we end up with a definition of those conditions needed to operate economically.

A good capability ratio is necessary to operate economically, but a good capability ratio is not sufficient to guarantee economical operation.

A predictable process is necessary to operate economically, but a predictable process is not sufficient to guarantee economical operation.

A way of operating a process on-target is necessary to operate economically, but operating on-target is not sufficient to guarantee economical operation.

All three of these necessary conditions must be present in order to guarantee economical operation. Do not focus on one condition while ignoring the other two.

This article and my two previous columns were excerpted from my new book ***Reducing Production Costs***, and are presented here with the permission of SPC Press. More complete explanations as well as tables for converting capability indexes into (1) *Effective Costs of Production and Use*, (2) *Excess Costs of Production*, and (3) *Excess Costs of Use* may be found there.