Is the Part in Spec?

How to create manufacturing specifications

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Over the past 20 years it has become fashionable to condemn measurement processes that are less than perfect. Yet the reality is that we must always use imperfect data. Given this fact of life, how can we ever know if a measured items is, or is not, within the specifications? Put another way, how can we make allowance for measurement error when characterizing product relative to specifications?

To complicate the answer to the questions above we have measurements which have measurement units attached and we have the uncertainty in those measurements which are expressed in terms of measurement units squared. Because of this complication, much of what has been written about this problem has been flawed. However, rigorous treatments of these issues are available, and in this article I shall present the results of these rigorous treatments.

THE UNCERTAINTY IN A MEASUREMENT

The uncertainty in any measurement may be defined by the standard deviation of repeated measurements of the same thing. In practice this quantity may be estimated by having one operator measure a set of things two or three times each using the same instrument as follows:

Product 42LF is made in small batches. The small batch size, along with the vigorous stirring which is part of the process, makes it reasonable to consider each batch as effectively homogenized. A single test sample is drawn from the batch while it is being stirred. This test sample is split in half and the viscosity is measured in duplicate. The test values, in centistokes, for seven batches are shown below:

Table 1: Duplicate Meaurements for Seven Batches

Batch	*78*	*79*	*80*	*81*	*82*	*83*	*84*
Viscosities	*2480*	*2870*	*2350*	*2990*	*3070*	*3020*	*2510*
	2530	*2730*	*2390*	*2930*	*3050*	*3000*	*2610*
Ranges	*50*	*140*	*40*	*60*	*20*	*20*	*100*

The duplicate measurements provide a simple and effective way of assessing the measurement error. The ranges shown above represent test-retest error of the same product, and therefore may be used to compute an estimate for the standard deviation of test-retest error:

$$\textit{Est. Std. Dev. for Test-Retest Error} \;=\; \frac{\textit{Average Range}}{d_2} \;=\; \frac{61.4}{1.128} \;=\; 54.4 \textit{ centistokes.}$$

When your boss asks what that means, what are you going to say? The value of 54.4 centistokes is an estimate of the square root of the rotational inertia about the averages for the set of test-retest values. Do you want to offer this as an explanation to your boss? If not, then you need a value that is easier to explain in terms of the problem, and in 1818 Wilhelm Bessel came up with just such a value: the Probable Error.

The Probable Error is simply 0.675 times the standard deviation of test-retest error. It defines the median amount by which a measurement will err: Half the time measurements of a known standard will differ from the accepted value by more than one Probable Error, and half the time these measurements will differ from the accepted value by less than one Probable Error.

Thus, the Probable Error defines the effective resolution of a measurement. If a measurement will differ from the "best value" by an amount greater than the Probable Error at least half of the time, then there will not be much point in attempting to interpret the number more precisely than plus or minus one Probable Error.

The Probable Error defines the essential uncertainty in the measurement. As long as the Probable Error is larger than the measurement increment, it will therefore define the effective resolution of a single value. Thus, the Probable Error defines the effective discreteness of a measurement, and it serves as a guide on how closely we should interpret a single value. For the example above our estimate for the Probable Error of the viscosity measurements would be:

Probable Error = 0.675 (54.4 cs.) = 36.7 centistokes.

The individual measurements of viscosity are recorded to the nearest 10 centistokes, but they are good to the nearest 37 centistokes. Half the time they will err by more than 37 centistokes, and half t he time they will err by less than 37 centistokes.

As soon as we know the Probable Error we also have an answer to the age-old question of how many digits to record. The measurement increment should be approximately the same size as the Probable Error:

Smallest Effective Measurement Increment = 0.2 Probable Error

Largest Effective Measurement Increment = 2 Probable Errors

When the measurement increment falls outside this range you will be recording too many or too few digits. (A rigorous explanation of this result is provided in Chapter 13 of my book *EMP III*: *Evaluating the Measurement Process and Using Imperfect Data*.) For the viscosity measurements the smallest effective measurement increment would be:

0.2 Probable Error = 0.2 (37cs.) = 7 centistokes

So that recording the viscosities to the nearest 10 centistokes is appropriate. Recording to the nearest centistoke would be excessive since the last digit recorded would be pure noise.

Likewise, rounding these measurements off to the nearest 100 centistokes would also be a mistake since the largest effective measurement increment here is 73 centistokes. Rounding to the nearest 100 centistokes would involve throwing away useful information.

WATERSHED SPECIFICATIONS

Since the Probable Error defines the effective resolution of a measurement, it will also define the effective increment to use in fuzzing up the specifications to make allowances for measurement error. However, before we begin to adjust our specifications we need to make allowance for the fact that specifications are generally stated in terms of acceptable values. That is, if I tell you the specifications are 6 to 12, you will understand that a value of 6 will be acceptable, as will a value of 12. If the measurement increment is whole numbers, the first unacceptable values will be

5 and 13. Since our computations are going to work with numbers as if they come from a continuum, we need to make a continuity correction and define the Watershed Specifications as:

Lower Watershed Specification = *Minimum Acceptable Value* – *One-half Measurement Increment.*

Upper Watershed Specification = *Maximum Acceptable Value* + *One-half Measurement Increment.*

Thus, when our specifications are 6 to 12, and parts are measured to the nearest whole number, our Watershed Specifications would be 5.5 to 12.5. With these values our specified tolerance would now be computed to be:

$$\textit{Specified Tolerance} = 12.5 - 5.5 = 7 \text{ units}$$

and our computation would now match the reality that there are seven acceptable values within the specifications of 6 to 12.

When our measurement increment is small relative to the specified tolerance, this adjustment will become trivial and can be ignored. However, as the measurement increment gets large relative to the specified tolerance this adjustment becomes essential for good computations.

FUZZING THE SPECIFICATIONS

The problem of using imperfect data is the problem of how to define manufacturing specifications such that, when a measurement falls within the manufacturing specifications the product is likely to be within the customer specifications. In figure 1 we see that there are two key aspects to this problem, the distance D and the definition of "likely."

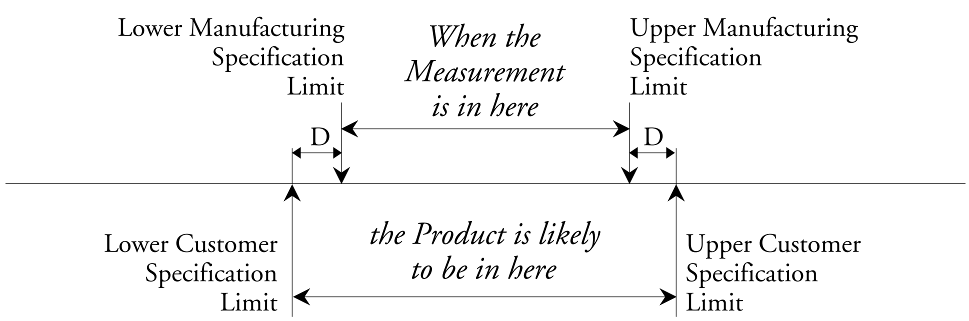

Figure 1: The Idea Behind Manufacturing Specifications

Since the Probable Error defines the effective resolution of a measurement it also defines the natural unit to use in fuzzing the specifications to allow for measurement error. In Chapter 14 of *EMP III* I go through the whole mathematical argument and obtain the following results.

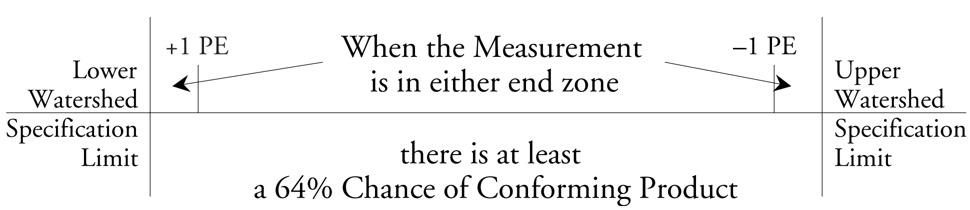

Figure 2: 64% Manufacturing Specifications = Watershed Specifications

If you use the watershed specifications as your manufacturing specifications, then (unless your measurement increment is too large) you will have items that will fall within one Probable Error of one of the watershed specifications at least part of the time. While you can say that these items have at least a 64% chance of being in spec, the fact that some of the items you are shipping have this small a chance of conformity means that the most you can say about your product stream is that it has at least a 64% of conforming product. Thus, when you use the watershed specifications you are effectively using 64% Manufacturing Specifications.

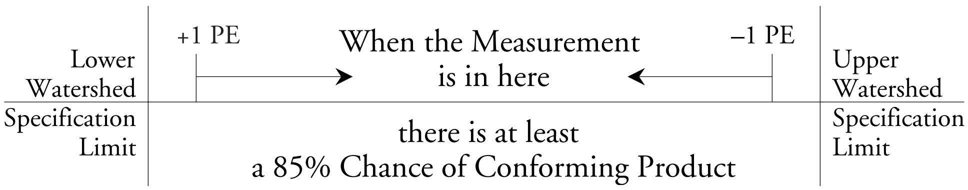

Figure 3: 85% Manufacturing Specifications

On the other hand, if you tighten the watershed specifications by one Probable Error on each end, then all of the product you will end up shipping will have at least an 85% chance of conforming. Thus, 85% manufacturing specifications are the watershed specifications tightened by one probable error.

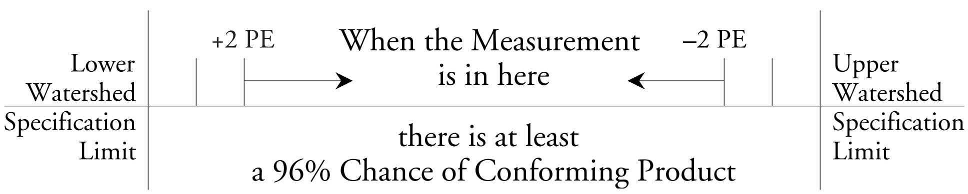

Figure 4: 96% Manufacturing Specifications

If you tighten the watershed specifications by two Probable Errors on each end, then all of the product you will end up shipping will have at least a 96% chance of conforming. Thus, 96% manufacturing specifications are the watershed specifications tightened by two probable errors.

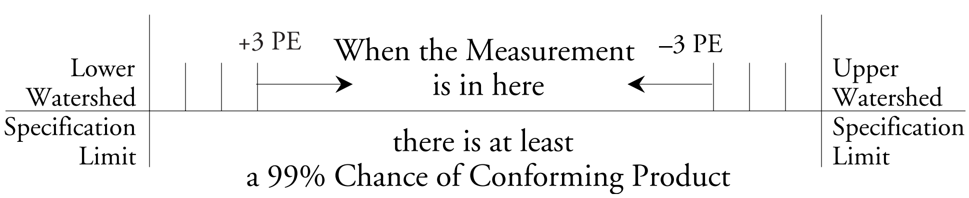

Figure 5: 99% Manufacturing Specifications

If you tighten the watershed specifications by three Probable Errors on each end, then all of the product you will end up shipping will have at least a 99% chance of conforming. Thus, 99% manufacturing specifications are the watershed specifications tightened by three probable errors.

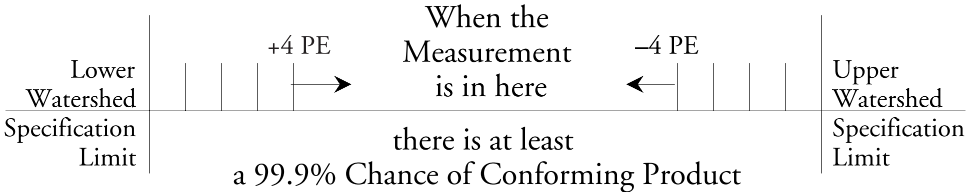

Figure 6: 99.9% Manufacturing Specifications

If you tighten the watershed specifications by four Probable Errors on each end, then all of the product you will end up shipping will have at least a 99.9% chance of conforming. Thus, 99.9% manufacturing specifications are the watershed specifications tightened by four probable errors.

The likelihoods cited above are *a posteriori* probabilities computed under the assumption that an unmeasured item had a 50-50 chance of conformity (hopefully you are doing better than this). This is why the likelihoods are given as minimums.

Given the trade-off between tighter specifications and higher likelihoods, and given that the likelihoods are minimums, I recommend using 96% manufacturing specifications in most situations.

Thus, once you have some knowledge about the test-retest error of your measurement system you can compute the Probable Error and use this value to determine the correct number of digits to record and to obtain manufacturing specifications that answer the perennial question, "How can we be sure that we are shipping conforming product?"